There is no Forward Guidance Puzzle*

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Abstract

According to the forward guidance puzzle, the economy responds more strongly to future than current interest rate cuts in many policy models used by central banks. We show that the puzzle arises only if three conditions are met: (1) a future interest rate cut is announced, (2) an interest rate peg precedes it, and (3) agents are assumed to attribute the peg exclusively to a sequence of monetary policy shocks. Absent any of these conditions, the response to a promised future interest rate cut weakens with the anticipation horizon, and forward guidance is no longer a forceful policy.

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1 Introduction

It is commonly believed that most policy models at central banks unreasonably predict that the economy responds strongly to announced policy changes far into the future. This belief arises from the so-called *forward guidance puzzle*, popularized by Del Negro, Giannoni, and Patterson (2023), who show that inflation responds stronger to a future interest rate change than a current one in a standard New Keynesian model.

We argue that this perceived puzzle is more fragile than it is usually made out to be. The result commonly described as puzzling arises only under a specific set of conditions: (1) a future interest rate cut is announced, (2) an interest rate peg precedes it, and (3) agents attribute the peg solely to a sequence of monetary policy shocks. The third condition is an assumption about the information that agents have, and it is rarely stated explicitly. Once it is relaxed, the puzzle is not an intrinsic feature of DSGE models but an artifact of implicitly assuming that the policy is generated by a series of sizable monetary policy shocks.

Consider first the case in which the central bank announces a future interest rate cut without a preceding peg. We prove that in any linearized general equilibrium model with a unique and stable equilibrium, the present response to a one-time future shock eventually decays as the shock is pushed further into the future. In this case the forward guidance puzzle cannot arise. We refer to this mechanism as *endogenous forward guidance*.

For example, under endogenous forward guidance, a central bank following a Taylor rule raises interest rates in response to rising output and inflation. When a future interest rate cut is announced, output and inflation are expected to increase in anticipation. The central bank counteracts this by raising interest rates before the cut takes effect. Hence, the central bank's endogenous response to its promise ensures that the promise itself has a muted impact on the economy.

In contrast, the standard forward guidance exercise—which we term *pegged forward guidance*—involves an announcement of a future interest rate cut while maintaining a pegged interest rate in all preceding periods. Under these conditions, the forward guidance puzzle may emerge: inflation rises in response to the announced rate cut and grows larger the further into the future the announcement extends.

We demonstrate that the key driver of this result is not the future rate cut itself, but rather the preceding interest rate peg and the series of monetary policy shocks it entails. Maintaining the peg requires a sequence of monetary policy shocks to prevent the central bank from adjusting rates according to its Taylor rule. Since the central bank would ordinarily raise interest rates to counteract the inflationary effects of the future cut, keeping rates pegged requires a series of expansionary monetary policy shocks. As the anticipation horizon increases, sustaining the peg demands increasingly large shocks, and these shocks—not the announced rate cut—generate the explosive inflation response. In this sense, the forward guidance puzzle is not a consequence of the policy announcement itself but of the monetary policy shocks required to uphold the preceding peg.

It is useful to be precise about the role of the zero lower bound here. In the experiments that generate the puzzle, the preceding peg is often motivated by a binding zero lower bound, following Eggertsson and Woodford (2003). What actually binds in the experiment, however, is an *upper* bound: in anticipation of the future cut the central bank would like to *raise* the interest rate, and it is the peg that prevents it from doing so. The mechanism behind the puzzle is thus driven by the constraint on rate *increases* ahead of the cut, not by the constraint on rate *decreases* that the zero lower bound represents. This is a complementary way of reading the same experiment, and it makes transparent why the size of the puzzle depends on how long the peg is sustained.

The main insight of this paper concerns the third condition. In the *pegged forward guidance* experiment, researchers implicitly assume that agents attribute the preceding peg solely to a sequence of monetary policy shocks. This is an assumption about agents' information: they are taken to know that no other shock is at work. To illustrate the importance of this assumption, consider the same pegged forward guidance setup (a future interest rate cut combined with a preceding peg), but have agents infer the most likely combination of shocks underlying the central bank's interest rate path. We extend the model to include both demand and monetary policy shocks and have agents observe only the interest rate path. The forward guidance puzzle then disappears: agents interpret the peg as caused by a combination of demand and monetary policy shocks, and because the demand shocks already account for much of the low interest rate, the monetary policy shocks needed to sustain the peg are small. We therefore conclude that there is no forward guidance puzzle in these models once agents are allowed to draw the most likely inference about the shocks they do not directly observe.

Related literature. This paper relates to the literature on the forward guidance puzzle. Del Negro et al. (2023) first documented this puzzling behavior and explained how it was related to passive monetary policy and indeterminacy in a standard New Keynesian model. Since the first version of Del Negro et al. (2023) in 2012, various studies have proposed explanations, including heterogeneity and incomplete markets (McKay, Nakamura, and Steinsson, 2016; Kaplan, Moll, and Violante, 2018; Hagedorn, Luo, Manovskii, and Mitman, 2019; Bilbiie, 2020), sticky information (Carlstrom, Fuerst, and Paustian, 2015;

Kiley, 2016), behavioral mechanisms (McKay, Nakamura, and Steinsson, 2017; Angeletos and Lian, 2018; Farhi and Werning, 2019; García-Schmidt and Woodford, 2019; Woodford, 2019; Gabaix, 2020), and credibility constraints (Bodenstein, Hebden, and Nunes, 2012; Andrade, Gaballo, Mengus, and Mojon, 2019; Campbell, Ferroni, Fisher, and Melosi, 2019).

The two most closely related papers are Maliar and Taylor (2019) and Eggertsson and Schüle (2024), both of which argue that there is no forward guidance puzzle. Maliar and Taylor (2019) conduct the same endogenous forward guidance exercise using the three-equation New Keynesian model and argue that forward guidance is ineffective away from the zero lower bound. Eggertsson and Schüle (2024) also emphasize that the preceding interest rate peg is necessary to generate the forward guidance puzzle, as it implies an implausibly large monetary policy regime shift.

Our analysis takes the central bank's policy rule as given and asks how the economy responds when that rule is perturbed. A complementary literature instead treats the interest rate path and inflation as the primitives and views the Taylor rule as an equilibrium-selection device whose coefficient is not separately identified (Cochrane, 2017, 2023; Werning, 2012; Woodford, 2012). From that vantage point, forward guidance at a peg or the zero lower bound is at heart a way of selecting an equilibrium, and the large anticipation effects reflect the explosive backward solution of the New Keynesian model. Our point is consistent with this reading: the explosive response in the pegged experiment is precisely the backward-explosive solution, and we show that it requires either a sequence of large monetary policy shocks or the assumption that agents attribute the peg solely to such shocks.

We make three contributions to the literature. First, as in Del Negro et al. (2023) and subsequent literature, we connect the forward guidance puzzle to passive monetary policy. We generalize this observation by proving that a one-time future shock cannot have an explosively growing present effect in any model that satisfies the Blanchard-Kahn conditions. This sharpens the point in Del Negro et al. (2023), whose Section II.C attributes the puzzle to passive policy, by showing that the result is a property of any determinate linearized model rather than of the three-equation model in particular. Second, we clarify the role of the zero lower bound: the puzzle is driven by the constraint on rate increases ahead of the cut, so the relevant bound in the experiment is an upper bound, not the lower bound that the zero lower bound represents. Third, and most importantly, we show that the puzzle requires the assumption that agents attribute the announced cut and preceding peg solely to monetary policy shocks. Once agents are allowed to infer the most likely combination of shocks behind the observed interest rate path, the puzzle disappears.

2 Forward guidance

We analyze the transmission of forward guidance using the standard three-equation New Keynesian model.

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \gamma(i_t - \mathbb{E}_t\{\pi_{t+1}\}) + z_t \tag{1}$$

$$\pi_t = \beta\mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t \tag{2}$$

$$i_t = \phi_\pi \pi_t + u_t^v \tag{3}$$

$$z_t = \rho_z z_{t-1} + u_t^z \tag{4}$$

$$u_t^z \sim N(0, \sigma_z) \tag{5}$$

$$u_t^v \sim N(0, \sigma_v) \tag{6}$$

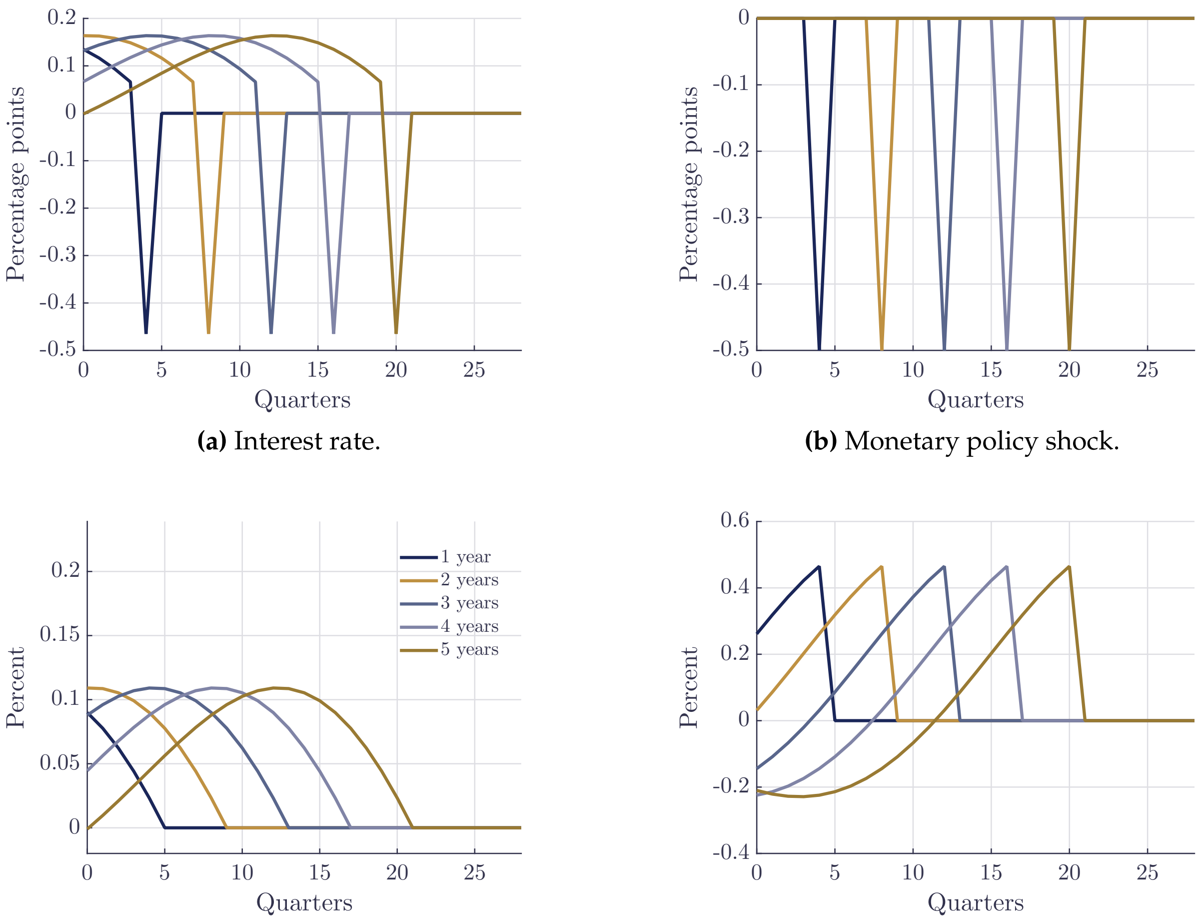
where y is the output gap, i is the nominal interest rate, π is the inflation rate, u^z is a demand shock, and u^v is a monetary policy shock. Equation (1) is the Euler equation where γ is the elasticity of intertemporal substitution, equation (2) is the Phillips curve where κ is its slope and β is the discount factor, and equation (3) is the Taylor rule with coefficient ϕ_π on inflation. We keep the model deliberately simple: the Taylor rule has neither interest-rate smoothing nor a response to output, and none of our results depends on adding either.[1] The parameter ρ_z is the persistence of the demand process. We use a standard calibration when we illustrate our arguments: $\gamma = 1$, $\beta = 0.99$, $\kappa = 0.05$, $\phi_\pi = 1.5$, and $\rho_z = 0.9$. The remaining two parameters, σ_z and σ_v, are the standard deviations of the two innovations. They play no role in the impulse responses and enter *only* the shock-inference exercise of Section 2.3, where they set the relative prior scale of the two shocks. We set $\sigma_z = 0.21$ and $\sigma_v = 0.18$.[2] The crucial assumption in Section 2.3 is that $\sigma_z > 0$.

2.1 Endogenous forward guidance

We first consider endogenous forward guidance, where the central bank announces a future interest rate cut and otherwise follows its Taylor rule. Figure 1 presents five separate experiments: in each, news arrives today (quarter 0) that the central bank will ease by 0.5 percentage points one, two, three, four or five years in the future. The key

[1] This responds to the point that a transparent theoretical argument should use the most stripped-down model that delivers it. Adding interest-rate smoothing or an output term leaves every conclusion below unchanged.

[2] Appendix B describes how the responses to anticipated shocks are computed. The simplified calibration plays the same role here as the estimated continuous-time model of the earlier draft (Ahn, 2023), but every parameter is now a textbook value.



(a) Interest rate. **(b)** Monetary policy shock.

(c) Inflation. **(d)** Output.

Notes: The figure displays five separate experiments. In each, news arrives today (quarter 0) that the central bank will ease monetary policy by 0.5 percentage points one, two, three, four or five years in the future, while otherwise following its Taylor rule. The interest rate rises ahead of the announced easing, and the present (quarter-0) response is smaller the further into the future the easing is announced.

Figure 1: Endogenous forward guidance.

result is that the present (quarter-0) inflation response is smaller the further into the future the easing is announced, and is already modest five years out.

The mechanism is visible in the interest-rate panel. When a future cut is announced, households increase consumption ahead of it, consistent with consumption smoothing, which raises output and inflation. The central bank, adhering to its Taylor rule, *raises* the interest rate in the interim, which dampens the effect of the future cut. The further into the future the cut is, the more this endogenous response attenuates its present impact.

The decay in Figure 1 is not special to this model. It holds in any linearized general equilibrium model with a unique and stable equilibrium, where the present response to a one-time future shock eventually decays to zero as the shock is moved further into the future.

Proposition 1. *Consider a linearized general equilibrium model that satisfies the Blanchard-Kahn conditions, so that the equilibrium exists and is unique. Let $V_0(k)$ denote the present response to a one-time shock realized k periods in the future. Then $\|V_0(k)\| \to 0$ as $k \to \infty$, at the rate given by the inverse of the smallest explosive eigenvalue of the system. If that eigenvalue is real, the decay is monotone. In general, the response decays in magnitude but may oscillate.*

We prove Proposition 1 in Appendix A.1. Under the Blanchard-Kahn conditions (Blanchard and Kahn, 1980), the present response to a shock k periods ahead is $(\Phi_{vv} - F\Phi_{gv})^{-k}F\varepsilon$, and the eigenvalues of $\Phi_{vv} - F\Phi_{gv}$ are exactly the explosive eigenvalues of the system (Theorem 1). Because these lie outside the unit circle, raising the matrix to the power $-k$ geometrically shrinks the response. However, the qualification in the Proposition matters. The proposition concerns the eventual magnitude of the response, not its monotonicity, since complex eigenvalues produce oscillatory decay. The claim is therefore that the present response cannot grow without bound in the horizon, which is what the forward guidance puzzle would require, not that it is monotone in every variable at every horizon.[3]

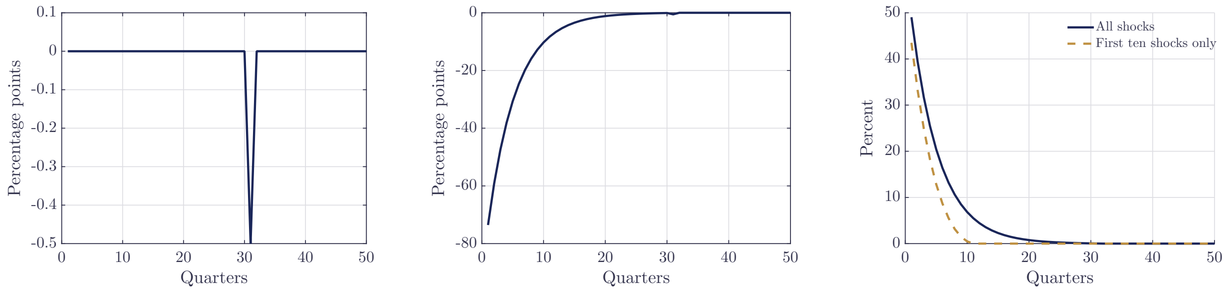
Maliar and Taylor (2019) make a related point: forward guidance is weak away from the zero lower bound. The interest-rate panel in Figure 1a clarifies why. The central bank responds to the announced future cut by *raising* interest rates before it happens, so the binding constraint in this experiment is an upper bound on the interest rate, not the lower bound. As we show next, it is exactly the imposition of the upper bound on the interest rate that revives the puzzle.

2.2 Pegged forward guidance

In contrast to the endogenous forward guidance above, the standard forward guidance exercise combines a promised future interest rate cut with a preceding interest rate peg. We refer to this policy as *pegged forward guidance*. This setup, illustrated in Figure 2a, is built in a model with only monetary policy shocks (we assume the standard deviation of the demand shock, σ_z is zero). Under these conditions, the forward guidance puzzle arises: the inflation response rises sharply going backward in time from the announced cut, and grows with the anticipation horizon.

The peg, however, is sustained by a sequence of expansionary monetary policy shocks, shown in Figure 2b. These shocks are constructed as the series of u^v's required to hold

[3] Figure 1c shows this concretely: the present inflation response is smaller the further into the future the easing is announced. With complex eigenvalues, the decay can be oscillatory rather than monotone, but what the proposition rules out is the explosive growth in the horizon that characterizes the puzzle.



(a) Interest rate. **(b)** Monetary policy shocks. **(c)** Inflation.

Notes: The figure displays the responses to an announced interest rate cut of 0.5 percentage points 30 quarters into the future, combined with a preceding interest rate peg, in a model with only monetary policy shocks ($\sigma_z = 0$). Panel (b) shows the sequence of monetary policy shocks required to sustain the peg. In Panel (c), the dashed line shows the inflation response when only the first ten of these shocks are included.

Figure 2: Pegged forward guidance. Only monetary policy shocks.

the interest rate flat. As the anticipation horizon extends, these shocks must become increasingly large to offset the central bank's usual response to expected inflation.

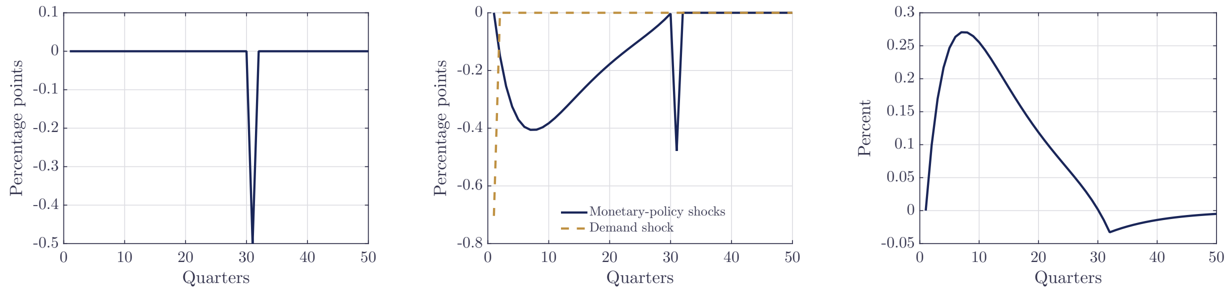
The intuition follows from comparing Figure 1a with Figure 2a: under its usual policy, the central bank would raise the interest rate ahead of the future cut to counteract an overheating economy. Since the peg prevents this, a sequence of increasingly expansionary monetary policy shocks is needed to sustain it. The longer the peg is sustained, the larger these shocks must be, and these shocks drive the inflation response.

Figure 2c makes this concrete: the dashed line shows the inflation response when only the first ten shocks are included, and it nearly coincides with the full response. Those initial shocks occur far from the announced cut, so the inflation explosion is driven by the shocks required to sustain the peg, not by the future interest rate cut.

2.3 Pegged forward guidance when agents infer shocks

The pegged forward guidance exercise assumed a model with only monetary policy shocks, so agents necessarily attributed the peg to a sequence of these shocks. Most models, however, contain several shocks, and an agent who observes the interest rate path but not its underlying causes must form a view about which shocks produced it. We now let agents do exactly that: holding the announced interest rate path fixed, we ask which combination of shocks is the most likely explanation, and trace the implied response.

Allowing for more than monetary policy shocks matters a great deal. As a first illustration, suppose agents read the peg as the consequence of one large contractionary



(a) Interest rate. **(b)** Shocks. **(c)** Inflation.

Notes: The figure displays the responses to the same interest rate path as Figure 2 (a peg followed by a 0.5 percentage point cut 30 quarters ahead), now generated from a combination of one contractionary demand shock and a sequence of monetary policy shocks.

Figure 3: Pegged forward guidance. One contractionary demand shock.

demand shock that pushes the economy into a recession, followed by a sequence of monetary policy shocks. Figure 3 shows the response under this interpretation. The interest rate path is identical to Figure 2, but the forward guidance puzzle is gone: inflation no longer explodes and instead stays within a fraction of a percent, orders of magnitude smaller than in Figure 2c. A single demand shock accounts for much of the low interest rate, so the monetary policy shocks needed to sustain the peg—and with them the inflation response—are now much smaller.

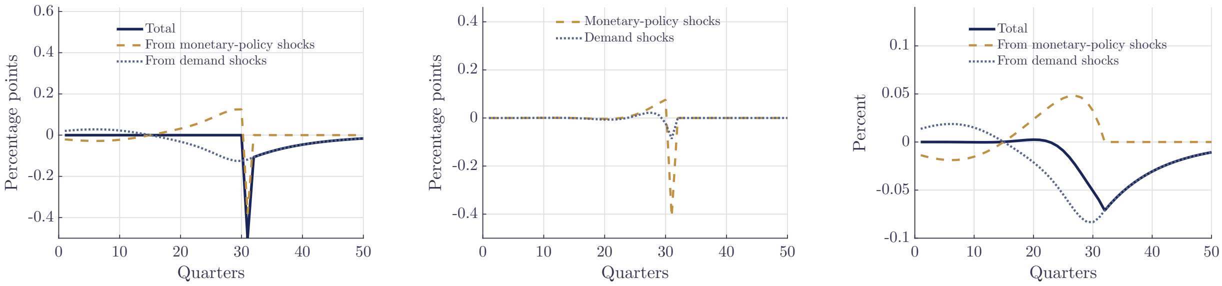
The example in Figure 3 fixes the shock composition ad hoc and is only meant to show that the interpretation matters. More generally, the same interest rate path can be produced by infinitely many combinations of shocks. A natural exercise is to let agents choose the most likely candidate. Agents observe only the announced interest rate path and, given the model and the prior distribution of shocks, compute the posterior over shock sequences consistent with that path, selecting the most likely (the conditional mean).[4]

Figure 4 reports the result in our model with demand and monetary policy shocks. The inflation response is no longer explosive. Agents read the peg as a mix of mostly contractionary demand shocks and expansionary monetary policy shocks whose effects on the interest rate exactly offset, as Panel (a) shows.[5] Because the two are inferred

[4] This is a one-shot inference problem: agents condition once on the announced path, using the model's shock distribution as a prior. It is not a model of adaptive learning, and so it differs from analyses in which agents gradually learn a possibly explosive law of motion, as in Christiano, Eichenbaum, and Johannsen (2018). Because the model is linear-Gaussian, the conditional mean is the standard Kalman-smoother formula, which we derive in Appendix C.

[5] Demand and monetary policy shocks are well suited to jointly explain a peg because they move the interest rate in the same direction while moving output and inflation in opposite directions, so they can

together rather than loaded entirely onto monetary policy, implied inflation stays close to zero. Hence, when agents are not forced into a single-shock interpretation, the forward guidance puzzle does not arise.



(a) Interest rate. **(b)** Inferred shocks. **(c)** Inflation.

Notes: The figure displays the responses to the same announced interest rate path as Figures 2 and 3 (a peg followed by a 0.5 percentage point cut 30 quarters ahead), when agents observe only the interest rate path and infer the most likely combination of demand and monetary policy shocks behind it. Panels (a) and (c) decompose the interest rate and inflation into the contributions of the two inferred shocks.

Figure 4: Pegged forward guidance. Agents infer shocks.

3 Conclusion

According to the forward guidance puzzle, the further into the future an interest rate cut is announced, the greater its immediate impact. We show that this result arises only when three conditions hold at the same time: (1) a future interest rate cut is announced, (2) an interest rate peg precedes it, and (3) agents attribute the peg exclusively to a sequence of monetary policy shocks.

The first two conditions describe the standard experiment. The third is an implicit assumption about agents' information that is rarely made explicit. With a determinate policy rule, a one-time future shock cannot have an explosively growing present effect, so the puzzle requires the peg, and the peg in turn requires either large monetary policy shocks or the assumption that agents attribute it solely to such shocks. Relax that assumption and let agents draw the most likely inference about the shocks, and the puzzle disappears. The forward guidance is thus less a property of the models central banks use, and more the result of one particular way of posing the policy experiment.

hold the interest rate flat without requiring large movements in either.

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Online Appendix for
"There is no Forward Guidance Puzzle"

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A Proofs

A.1 Proof of Proposition 1

Suppose the linearized system of equations for a general equilibrium model satisfying uniqueness is given by[1]

$$\begin{bmatrix} V_{t+1} \\ g_{t+1} \end{bmatrix} = \Phi \begin{bmatrix} V_t \\ g_t \end{bmatrix} + \begin{bmatrix} 0 \\ \Psi \end{bmatrix} \varepsilon_{t+1} \equiv \begin{bmatrix} \Phi_{vv} & \Phi_{vg} \\ \Phi_{gv} & \Phi_{gg} \end{bmatrix} \begin{bmatrix} V_t \\ g_t \end{bmatrix} + \begin{bmatrix} 0 \\ \Psi \end{bmatrix} \varepsilon_{t+1} \tag{A.1}$$

where V denotes the forward-looking variables, g the pre-determined variables, and ε_{t+1} shocks satisfying $\mathbb{E}_t[\varepsilon_{t+1}] = 0$.Let F be the solution (decision rule) of the system, i.e., $\begin{bmatrix} F \\ I \end{bmatrix}$ forms the basis vectors for the stable subspace of Φ, where I is the identity matrix.

Consider the problem for a shock $\varepsilon_{k|0}$ that will be realized at time k but known at time 0. Given the linear form, we can substitute forward to get

$$\begin{bmatrix} V_k \\ g_k \end{bmatrix} = \begin{bmatrix} \Phi_{vv} & \Phi_{vg} \\ \Phi_{gv} & \Phi_{gg} \end{bmatrix}^k \begin{bmatrix} V_0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{k|0} \end{bmatrix} \tag{A.2}$$

Once the shock is realized, the system must return to the stable subspace to satisfy the transversality condition. Hence, we have

$$V_k = (\Phi^k)_{vv} V_0 = F g_k = F(\Phi^k)_{gv} V_0 + F\varepsilon_{k|0} \tag{A.3}$$

resulting in

$$V_0 = \left[(\Phi^k)_{vv} - F(\Phi^k)_{gv} \right]^{-1} F\varepsilon_{k|0}. \tag{A.4}$$

[1]Expectation operators are ignored for simplicity of notation. Expectation operators will just remove the shock-terms (certainty equivalence).

We can further simplify the expression using the relationship in Lemma 1.

Lemma 1.

$$(\Phi^k)_{vv} - F(\Phi^k)_{gv} = (\Phi_{vv} - F\Phi_{gv})^k \tag{A.5}$$

Proof. See Appendix A.2. □

Plugging (A.5) into (A.4) gives

$$V_0 = (\Phi_{vv} - F\Phi_{gv})^{-k} F\varepsilon_{k|0}. \tag{A.6}$$

Lemma 1 shows that the component in the orthogonal complement of the stable subspace evolves by the dynamics projected on to the orthogonal complement of the stable subspace. In economic terms: the difference between the planned behavior with and without anticipated shocks remains orthogonal to the planned behavior without anticipated shocks under the system dynamics.

As the initial response satisfies (A.6), it is sufficient to analyze the eigenvalues of $(\Phi_{vv} - F\Phi_{gv})$ to understand whether present responses to future shocks strengthen or weaken with the anticipation horizon. Because the matrix multiplication involves an inverse, the present response to a future shock will weaken for eigenvalues outside the unit circle. One could get the impression that it is possible to have present responses become stronger in the anticipation horizon from reading the literature on the forward guidance. However, this is not possible for a general equilibrium model with a unique equilibrium.

Theorem 1. *For a linearized general equilibrium model of the form (A.1), if the equilibrium exists and is unique for all possible initial condition g_0, then the eigenvalues of $(\Phi_{vv} - F\Phi_{gv})$ are equal to the eigenvalues of Φ outside the unit circle.*

Proof. See Appendix A.3. □

Theorem 1 can be summarized as Proposition 1: in any linearized general equilibrium models with a unique equilibrium, any present response to a one-time shock in the future becomes weaker the further away the shock is.

A.2 Proof of Lemma 1

Proof. For any g, $\begin{bmatrix} Fg \\ g \end{bmatrix}$ is on the stable manifold by definition of F. Hence,

$$\Phi \begin{bmatrix} Fg \\ g \end{bmatrix} = \begin{bmatrix} \Phi_{vv}Fg + \Phi_{vg}g \\ \Phi_{gv}Fg + \Phi_{gg}g \end{bmatrix} \tag{A.7}$$

is also on the stable manifold by the invariance of the stable manifold.[2] Using the definition of F again,

$$\Phi_{vv}Fg + \Phi_{vg}g = F\Phi_{gv}Fg + F\Phi_{gg}g \tag{A.8}$$

holds for all g. Hence, we have

$$\Phi_{vv}F + \Phi_{vg} = F\Phi_{gv}F + F\Phi_{gg}. \tag{A.9}$$

Moreover, we have

$$\begin{bmatrix} (\Phi^k)_{vv} \\ (\Phi^k)_{gv} \end{bmatrix} = \begin{bmatrix} \Phi_{vv} & \Phi_{vg} \\ \Phi_{gv} & \Phi_{gg} \end{bmatrix} \begin{bmatrix} (\Phi^{k-1})_{vv} \\ (\Phi^{k-1})_{gv} \end{bmatrix} = \begin{bmatrix} \Phi_{vv}(\Phi^{k-1})_{vv} + \Phi_{vg}(\Phi^{k-1})_{gv} \\ \Phi_{gv}(\Phi^{k-1})_{vv} + \Phi_{gg}(\Phi^{k-1})_{gv} \end{bmatrix}. \tag{A.10}$$

Finally, we can prove our relationship. Consider an induction step for a given k,

$$\begin{aligned} (\Phi^k)_{vv} - F(\Phi^k)_{gv} &= \left[\Phi_{vv}(\Phi^{k-1})_{vv} + \Phi_{vg}(\Phi^{k-1})_{gv}\right] - F\left[\Phi_{gv}(\Phi^{k-1})_{vv} + \Phi_{gg}(\Phi^{k-1})_{gv}\right] && \text{(A.11)} \\ &= (\Phi_{vv} - F\Phi_{gv})(\Phi^{k-1})_{vv} - (F\Phi_{gg} - \Phi_{vg})(\Phi^{k-1})_{gv} && \text{(A.12)} \\ &= (\Phi_{vv} - F\Phi_{gv})(\Phi^{k-1})_{vv} - (\Phi_{vv} - F\Phi_{gv})F(\Phi^{k-1})_{gv} && \text{(A.13)} \\ &= (\Phi_{vv} - F\Phi_{gv})^k && \text{(A.14)} \end{aligned}$$

where the second equality comes from substituting in the invariance condition (A.9), and the last equality from induction. □

[2] Alternately, the intuitive statement is that if the jump variables, V, are determined by the solution (decision rule) F, then the values in the next period need to satisfy the decision rule relationship as well. The fact that such a rule F exists comes from the invariance of the stable manifold, not the other way.

A.3 Proof of Theorem 1

Proof. Denote a Jordan normal form[3] of

$$\begin{bmatrix} \Phi_{vv} & \Phi_{vg} \\ \Phi_{gv} & \Phi_{gg} \end{bmatrix} \tag{A.15}$$

by

$$\begin{bmatrix} \Phi_{vv} & \Phi_{vg} \\ \Phi_{gv} & \Phi_{gg} \end{bmatrix} = P \begin{bmatrix} T_u & 0 \\ 0 & T_s \end{bmatrix} P^{-1} \tag{A.16}$$

where

$$P \equiv \begin{bmatrix} P_{vu} & P_{vs} \\ P_{gu} & P_{gs} \end{bmatrix} \tag{A.17}$$

where the eigenvalues are ordered such that T_u contains the explosive eigenvalues and T_s the stable eigenvalues.[4] From the existence and uniqueness requirement, we have the correct number of stable and explosive eigenvalues for T_u and T_s. Further, we get the 0 in the upper-right and lower-left corners of the (block) Jordan normal form as T_u and T_s cannot share eigenvalues.

From existence and uniqueness, P_{gs} is invertible and we can solve for F with our Jordan normal form with invertibility. With this Jordan normal form, we have that $\begin{bmatrix} P_{vs} \\ P_{gs} \end{bmatrix}$ form the basis vectors for the stable subspace. Hence, we have $\begin{bmatrix} P_{vs}P_{gs}^{-1} \\ I \end{bmatrix}$ also forms the basis vectors for the stable subspace, i.e.,

$$F \equiv P_{vs}P_{gs}^{-1}. \tag{A.18}$$

[3]Computation of the Jordan normal form is not numerically stable, so the Jordan normal form is only used for the proof.

[4]One can trivially reorder any Jordan normal form to get the required ordering of eigenvalues.

Invertibility of P_{gs} further gives us the block matrix inverse formula of

$$P^{-1} = \begin{bmatrix} \widetilde{P}_{vu} & \widetilde{P}_{vs} \\ \widetilde{P}_{gu} & \widetilde{P}_{gs} \end{bmatrix} \tag{A.19}$$

$$\widetilde{P}_{vu} \equiv \left(P_{vu} - P_{vs}P_{gs}^{-1}P_{gu}\right)^{-1} \tag{A.20}$$

$$\widetilde{P}_{vs} \equiv -\left(P_{vu} - P_{vs}P_{gs}^{-1}P_{gu}\right)^{-1} P_{vs}P_{gs}^{-1} \tag{A.21}$$

$$\widetilde{P}_{gu} \equiv -P_{gs}^{-1}P_{gu}\left(P_{vu} - P_{vs}P_{gs}^{-1}P_{gu}\right)^{-1} \tag{A.22}$$

$$\widetilde{P}_{gs} \equiv P_{gs}^{-1} + P_{gs}^{-1}P_{gu}\left(P_{vu} - P_{vs}P_{gs}^{-1}P_{gu}\right)^{-1} P_{vs}P_{gs}^{-1} \tag{A.23}$$

Multiplying out (A.16) with our notation, we get

$$\Phi_{vv} - F\Phi_{gv} = \left(P_{vu}T_u\widetilde{P}_{vu} + P_{vs}T_s\widetilde{P}_{gu}\right) - F\left(P_{gu}T_u\widetilde{P}_{vu} + P_{gs}T_s\widetilde{P}_{gu}\right) \tag{A.24}$$

$$= (P_{vu} - P_{vs}P_{gs}^{-1}P_{gu})T_u\widetilde{P}_{vu} \tag{A.25}$$

$$= (P_{vu} - P_{vs}P_{gs}^{-1}P_{gu})T_u(P_{vu} - P_{vs}P_{gs}^{-1}P_{gu})^{-1} \tag{A.26}$$

The last line is a Jordan normal form of $(\Phi_{vv} - F\Phi_{gv})$, and hence, the eigenvalues of $(\Phi_{vv} - F\Phi_{gv})$ are precisely the explosive eigenvalues of the original system. □

B Computation of the figures

This appendix describes how the responses in the figures are computed. The only non-standard step is computing the response to an *anticipated* shock—one that is known at date 0 but realized at a future date. We do this by solving a single stacked linear system rather than by an augmented-state or Blanchard-Kahn routine, which keeps the computation transparent. Substituting the Taylor rule (3) into the Euler equation (1), the model over dates $t = 1, \dots, T$ is

$$y_t - y_{t+1} + \gamma\phi_\pi \pi_t - \gamma\pi_{t+1} = -\gamma u_t^v + z_t, \tag{A.27}$$

$$\pi_t - \beta\pi_{t+1} - \kappa y_t = 0, \tag{A.28}$$

with the terminal condition $y_{T+1} = \pi_{T+1} = 0$, which for T large selects the unique stable equilibrium. Stacking these $2T$ equations in the unknowns $(y_1, \dots, y_T, \pi_1, \dots, \pi_T)$ gives a square linear system $Ax = c$, where the right-hand side c is linear in the shock paths $\{u_t^v\}$ and $\{z_t\}$. Inverting A once yields the linear operators that map any shock path into the equilibrium paths of y, π, and $i = \phi_\pi\pi + u^v$. The columns of these operators are the anticipated-shock impulse responses used in Figure 1.

The pegged experiments invert the same operators. To generate Figures 2 and 3 we fix a target interest rate path (a peg at zero for thirty quarters, then a 0.5 percentage point cut) and solve for the shocks that reproduce it: with monetary policy shocks only, we solve $M_i u^v = \bar{\imath}$ for u^v, where M_i is the operator mapping monetary policy shocks into the interest rate and $\bar{\imath}$ is the target path. For Figure 3 the first shock is instead a demand shock and the rest are monetary policy shocks. The shock-inference exercise of Figure 4 uses the same operators inside the Gaussian conditioning formula derived in Appendix C.

C Conditional Probability

For our linear model with the given shock structure, the resulting distribution is Gaussian, and the computation of the conditional probability is equivalent to computing the conditional distribution of a normal distribution. This is standard and only presented for completeness.

Suppose we have exogenous shocks and states with uncertainty collected as x

$$x \sim N(\mu_x, \Sigma_x) \tag{A.29}$$

with observation given as

$$y = Hx. \tag{A.30}$$

If we stack x and y, then we get that

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_x \\ H\mu_x \end{bmatrix}, \begin{bmatrix} \Sigma_x & \Sigma_x H' \\ H\Sigma_x & H\Sigma_x H' \end{bmatrix}\right). \tag{A.31}$$

Hence, we can compute the conditional probability given an observation of $y = a$ as

$$(x|y = a) \sim N\left(\overline{\mu}, \overline{\Sigma}\right), \tag{A.32}$$

$$\overline{\mu} = \mu_x + \Sigma_x H'(H\Sigma_x H')^{-1}\left(a - H\mu_x\right), \tag{A.33}$$

$$\overline{\Sigma} = \Sigma_x - \Sigma_x H'(H\Sigma_x H')^{-1} H\Sigma_x. \tag{A.34}$$

One can refer to section 4.13 of Durbin and Koopman (2012) to see that this is an equivalent formulation for Kalman smoothing.