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Consumption with liquidity constraints: An analytical characterization

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ABSTRACT

How do liquidity constraints affect households? This is a well-researched subject with remarkably few theoretical results. This paper bridges this gap by providing a closed form expression for consumption with liquidity constraints for a wide class of utility functions (HARA). Using this expression, I prove that the consumption function is strictly concave in wealth if a relevant liquidity constraint exists. Moreover, households respond to a tightening of the liquidity constraint by reducing consumption, becoming more sensitive to wealth changes, and having a "more" concave consumption function in wealth. These results thus provide theoretical underpinnings for how contractions in credit supply affect households.

1. Introduction

The question of how liquidity constraints affect households attracts a lot of attention. Numerous important papers discuss how credit constraints affect consumption, but despite the widespread interest, there are remarkably few analytical results on liquidity constraints. This paper bridges this gap by providing analytical results on two important questions: How does the presence of a liquidity constraint affect household behavior? And how does changes in liquidity constraints affect household behavior?

To address these questions, I study a consumption-saving problem with liquidity constraints for a wide class of utility functions.[1] The key to my results relies on the observation that the household problem with a liquidity constraint is a solvable first-order differential equation in optimal consumption. I derive this implicit closed form expression of the consumption function where the liquidity constraint enters as a boundary constraint. Using this framework, I prove how the presence of and shifts in liquidity constraints affect consumption.

First, I compare the consumption function with and without a liquidity constraint, and show that households respond to the constraint by reducing consumption, becoming more sensitive to wealth changes, and having a consumption function that is strictly concave in wealth. The intuition is as follows: the constraint forces households to reduce consumption at the constraint. Due to consumption smoothing, households also reduce consumption for other wealth levels, but this effect is weaker as the agent becomes richer and moves away from the constraint. The constraint thus makes households more sensitive to wealth changes, and the gradual reduction in the effect of the constraint introduces a concavity in the consumption function.

Second, I show that households respond to a tighter liquidity constraint by reducing consumption, becoming more sensitive to wealth changes, having a "more" concave consumption function in wealth. This result thus provides the theoretical underpinnings of the effects of contractions in credit supply on households.

Although this paper is not the first to characterize how liquidity constraints affect consumption, it is the first to provide an analytical solution to the consumption function with liquidity constraints. For example, Seater (1997) and Park (2006) use a continuous time approach similar to my paper. However, Seater (1997) restricts his analysis to cases where time-discounting equals the interest rate, while Park (2006) restricts his analysis to CRRA utility. My paper generalizes both these results by providing the effects of liquidity constraints on consumption for a general class of utility functions. Carroll and Kimball (2001) analyzes a discrete time version of the model and find that the introduction of a liquidity constraint makes the modified consumption function 'more' concave ('counterclockwise concavification'). However, Carroll and Kimball (2001) restricts the analysis to finite life-time and period-for-period constraints while I consider the case with infinitely-lived households and a constraint that matters in each period. Further, Nishiyama and Kato (2012) show that the consumption function is concave in wealth with quadratic utility if a liquidity constraint exists,

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[1] I study the HARA-class (Hyperbolic Absolute Risk Aversion), containing common utility functions in macroeconomics such as the power, exponential, and quadratic utility functions.

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implying that the effects of the liquidity constraint is independent of prudence. I show that this result holds generally: the liquidity constraint makes optimal consumption concave even if the utility function exhibit negative prudence.

2. The model

Households are infinitely-lived and maximized its discounted utility flow from consumption

$$\int_t^\infty e^{-\rho s} u(c(w_s))ds \tag{1}$$

where $u(c)$ is a utility function of the HARA class

$$u(c) = \begin{cases} \frac{1}{a-1}(ac+b)^{1-1/a} & a \neq 0, 1 \\ -be^{-c/b} & a = 0 \\ \log(c+b) & a = 1 \end{cases} \tag{2}$$

with $b > \max\{-ac, 0\}$. Note that $u'(c) > 0$ and $u''(c) < 0$ always hold for HARA utility. Household wealth takes the form of risk-free bonds and evolves according to

$$dw_t = (rw_t + y - c(w_t))dt \tag{3}$$

where r is the real interest rate and y is constant income. The household also faces an exogenous liquidity constraint on wealth holdings

$$w_t \geq \underline{w} \tag{4}$$

where $\underline{w}$ is a scalar satisfying $\underline{w} > -\frac{y}{r}$ (natural borrowing limit). The following Hamilton–Jacobi–Bellman (HJB) equation describes the solution to the household problem[2]

$$\rho V(w) = \max_c u(c) + V'(w)(rw + y - c) \tag{5}$$

where $V(w)$ is the value function of a household with assets w. The first order necessary condition is

$$u'(c) = V'(w). \tag{6}$$

To obtain a differential equation in consumption, apply the envelope theorem on Eq. (5), use the first order condition to replace all value functions with utility functions, and use the fact that $-\frac{u'(c(w))}{u''(c(w))} = ac(w) + b$ for HARA utility functions to obtain

$$c(w) = rw + y + \frac{ac(w)+b}{c'(w)}(\rho - r). \tag{7}$$

The optimal solution to the household problem is therefore a first-order non-linear differential equation in consumption.

3. Consumption with liquidity constraints

This section contains two main results. First, I show that the differential equation is solvable and I obtain an implicit expression (8) for optimal consumption in Proposition 1. Using expression (8), it is straightforward to derive three results on the qualitative properties of consumption with liquidity constraints in Proposition 2.

Proposition 1. *Suppose $\underline{w} > -\frac{y}{r}$, $\rho > r > 0$, then*

$$c(w) = (rw + y)\left(\frac{r + a(\rho - r)}{r}\right) + b\frac{\rho - r}{r} - \frac{\rho - r}{r}\left(a(r\underline{w} + y) + b\right)\left(\frac{a(r\underline{w} + y) + b}{ac(w) + b}\right)^{\frac{r}{a(\rho - r)}} \tag{8}$$

optimally solves (7) for all HARA utility functions (2).

Proof. See Appendix A.1. □

Proposition 1 provides an implicit expression for optimal consumption with a liquidity constraint. By comparing consumption in the constrained and the unconstrained case, I can characterize what happens to optimal consumption when I add a liquidity constraint

Proposition 2. *For utility functions of the HARA class, if $\underline{w} > -\frac{y}{r}$, $\rho > r > 0$, and $r + a(\rho - r) > 0$. Then there exists an optimal consumption function for all $w \in (\underline{w}, \infty)$ where*

1. *Consumption is lower than in the unconstrained case: $c_u(w) > c(w) > rw + y$.*
2. *The MPC out of wealth is greater than in the unconstrained case: $c'(w) > c_u'(w) > 0$.*
3. *Consumption is strictly concave in wealth: $c''(w) < 0$.*
4. *If also $a > 0$, consumption is asymptotically linear: $\lim_{w\to\infty} c(w) = c_u(w)$.*

Proof. See Appendix A.2. □

Proposition 2 presents four effects of introducing a liquidity constraint in the optimal consumption problem. First, the presence of a liquidity constraint reduces consumption for all levels of wealth. The liquidity constraint imposes a shadow cost on consumption because households know that the constraint binds at some point in the future. Due to the intertemporal substitution motive, households become more reluctant to consume and reduce their optimal consumption. Second, the MPC out of wealth is always greater with a liquidity constraint than in the unconstrained case. Since households want to avoid the liquidity constraint, each unit of wealth is more valuable to households. They are therefore more sensitive to wealth changes, increasing their MPC out of wealth. Third, the optimal consumption function is strictly concave in wealth in the presence of a liquidity constraint. The effects of the liquidity constraint are greatest at the constraint, but also affects all wealth levels through the intertemporal substitution motive. However, the effect is declining in the distance from the constraint. Poor households reduce consumption more than wealthy households. This asymmetric reduction in consumption results in an optimal consumption function that is strictly concave in wealth. Fourth, if utility also satisfies decreasing absolute risk aversion (DARA), consumption is asymptotically linear as wealth goes to infinity. However, note that the optimal consumption function is never linear for a finite value of wealth since $c_u(w) > c(w)$.

Remark 1. The results in Proposition 2 only hold for $w \in (\underline{w}, \infty)$. For $w = \underline{w}$, $c(\underline{w}) = r\underline{w} + y$ while $c'(\underline{w})$ and $c''(\underline{w})$ are undefined.

Remark 2. The condition $\underline{w} > -\frac{y}{r}$ ensures that the liquidity constraint is tighter than the natural borrowing limit.

Remark 3. The results in Proposition 2 are independent of the third derivative of the utility function.

Remark 4. The condition $\rho > r > 0$ ensures that the liquidity constraint is relevant.

Remark 5. The condition $r + a(\rho - r) > 0$ ensures that $c'(w) > 0$ for all $w \in (\underline{w}, \infty)$.

4. Shifts in the liquidity constraint

In the previous section, I compared the consumption function in the unconstrained case and the case where there is a relevant constraint. In this section, I extend the results in Proposition 2 to

[2] See Chang (2004) or Achdou et al. (2017) for how to derive this expression.

the case where I compare an original consumption function with a liquidity constraint to a modified consumption function with a tighter constraint.

Proposition 3 (*The Effects of a Tighter Liquidity Constraint*)**.** *For utility functions of the* DARA *class ($a > 0$), if $u' > 0$, $u'' < 0$, $\underline{w} > -\frac{y}{r}$, $\rho > r > 0$, and $r + a(\rho - r) > 0$. Then for all $w \in (\underline{w}, \infty)$, the effects of a tightening of the liquidity constraint ($\tilde{\underline{w}} > w$) on consumption are*

1. *Consumption decreases: $c(w; \tilde{\underline{w}}) < c(w; \underline{w})$.*
2. *The* MPC *out of wealth increases: $c'(w; \tilde{\underline{w}}) > c'(w; \underline{w})$.*
3. *The consumption function is "more" concave in wealth: $c''(w; \tilde{\underline{w}}) < c''(w; \underline{w})$.*

Proof. See Appendix A.3. □

Proposition 3 presents three qualitative effects of tighter credit conditions on household behavior. consumption decreases, the MPC out of wealth increases, and consumption is "more" concave in wealth. The results in Proposition 3 thus imply that a tighter liquidity constraint always strengthen the effects of the liquidity constraint, inducing a greater deviation from the unconstrained (linear) case.

5. Conclusion

In this paper, I derive an analytical expression for optimal consumption for households with HARA utility, and show that the introduction of a liquidity constraint reduces consumption, increases the marginal propensity to consume out of wealth, and makes the consumption function concave in wealth. Further, for households with DARA utility, a tighter liquidity constraint induces households to reduce consumption, increase their marginal propensities to consume, and make the consumption function "more" concave in wealth.

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Appendix A. Proofs

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2018.03.004.

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