The Effect of Capacity Constraints on the Slope of the Phillips Curve*

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December 2023

Abstract

How do capacity constraints affect the slope of the Phillips curve? In particular, do changes in output affect prices by more when firms face more strongly diminishing returns? We investigate this question in a New Keynesian framework. Intuitively, one may conjecture that the answer is yes. With more strongly diminishing returns, marginal costs increase more when production increases, also raising prices by more. The Phillips curve, the relationship between output and inflation, should therefore be *steeper* with more strongly diminishing returns. But this intuition is incomplete in models with sticky prices à la Calvo. In the standard New Keynesian model with Calvo pricing frictions, more strongly diminishing returns actually flatten the Phillips curve, given the standard parameterisation of the model. The reason is the reluctance of firms who alter their prices to let their prices get too far out of line with the prices set by firms that do not adjust. We explain how the Phillips curve might be amended to avoid this implausible feature, for example by using Rotemberg pricing or introducing wage stickiness.

JEL: E52

Keywords: Monetary Policy, the Phillips Curve

*We are grateful to Ken Mayhew and two anonymous referees for extremely helpful comments on earlier versions of this paper.

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1 Introduction

Recently, inflation has resurfaced as a major concern in many western countries. High energy prices, supply problems, and high aggregate demand have resulted in inflation numbers not seen since the 1970s. This paper explores how changes in aggregate demand affect prices in a situation with capacity constraints or supply bottlenecks. More concretely, we ask how more strongly diminishing returns in production affect the slope of the Phillips curve.

This question is important for understanding the inflation process under supply constraints. Our study is motivated by the recent inflation episode where exogenously imposed restrictions (e.g., Covid-related lockdowns) adversely affect supply - which might affect inflation dynamics. However, our results are also relevant for more general episodes, such as when supply contracts endogenously, for example, due to hysteresis effects on labor supply in the aftermath of recessions.

Informally, one may conjecture that more strongly diminishing returns in production strengthen the effects of aggregate demand on prices. Consider a model in which firms compete under monopolistic competition. In such models, firms set prices P as a markup $\mathcal{M}$ over nominal marginal costs

$$P_t = \mathcal{M} \cdot \frac{W_t}{MPN_t} \tag{1}$$

where nominal marginal costs are equal to the nominal wage W divided by the marginal product of labour MPN. First of all, higher output directly affects wages, resulting in higher marginal costs and prices, and then, secondly, when there are diminishing returns, the marginal product of labour also falls as output increases, raising marginal costs further. More strongly diminishing returns would seem to strengthen this second effect, therefore meaning that the Phillips curve becomes steeper.

But this intuition is incomplete when firms face sticky prices à la Calvo. As we shall see, in the standard New Keynesian model (Galí, 2015, Chapter 3), more strongly diminishing returns actually flatten the Phillips curve for all reasonable model calibrations. In the model, only some firms are visited by the Calvo fairy in each period and can raise their prices following an increase in aggregate demand, what we can call the "price adjusters." But other firms, the "non-adjusters," do not do this. As a result of this change in the relative price of the two sets of firms, the demand for the output of the adjusting firms will fall relative to that of the non-adjusters, although – nevertheless – the output of both kinds of firms will increase following the increase in aggregate demand. This reduction in relative demand, and output, of the adjusting firms results in a higher marginal product of labour and, thus, a lower marginal cost for the adjusters, reducing the price which they will choose to set relative to what would have happened if all firms had been free to increase their prices at the same time in response to the increase in aggregate demand. This relative price effect will tend to flatten the Phillips curve.

When will this relative price effect channel dominate in determining the effect of more strongly diminishing returns on the slope of the Phillips curve? The answer is that this will happen if the real wage increase in response to a demand change is sufficiently large.

To see this, first assume the opposite, namely that real wages do not respond to demand. In this case, a change in output only affects prices via its effect on the marginal product of labour in price-adjusting firms. In the extreme case where the marginal product of labour also does not respond to output (i.e., the production function is linear in labour), output also does not affect prices, and the Phillips curve will be flat. As we introduce diminishing returns, we open a channel from output to prices. Moreover, because (by assumption) the only channel from output to prices is via diminishing returns in the price adjusting firms, more strongly diminishing returns will steepen the Phillips curve: an adjusting firm that is facing a more sharply rising marginal cost curve because of the stronger diminishing returns will both increase its prices by more, and reduce its output by more (relative to the output of the non-adjusters), than a firm with less strongly diminishing returns.

Now consider the case where the economy-wide increase in the real wage is large. An adjusting firm will then face an increase in its marginal costs caused by factors largely outside its control, namely the increase in the real wage caused by the increase in the output of all firms. Adjusters will, of course, also face pressure to increase their prices as they increase their output because they are subject to diminishing returns for the reasons explained above. But the main reason they will face pressure to increase their prices is the externally-imposed increase in their costs coming from the wage increase. As noted earlier, as adjusters raise prices, they reduce their output relative to that of the non-adjusters. This relative reduction in output reduces their need to increase prices in the first place because their marginal product of labour increases. More strongly diminishing returns will strengthen this feedback channel. And since the feedback channel primarily works by dampening the effect of an economy-wide wage increase on prices, the feedback channel will be important in settings where the economy-wide wage increase is large. Indeed, this second effect will dominate for standard parameterisations of the Calvo Phillips curve.

This result appears to be a theoretical curiosity - an oddity coming from the assumption of time-dependent pricing embedded within the Calvo Phillips curve. There are two ways to obtain the perhaps more intuitive result that more strongly diminishing returns will steepen the Phillips curve. The first is not to use a time-dependent pricing model. The above result stems from a feature of time-dependent pricing in which the increase in aggregate demand leads to price dispersion because only some firms adjust their prices. Alternative models will remove or dampen this feedback channel. For example, the feedback channel is absent in Rotemberg models with convex adjustment costs because an increase in aggregate demand does not cause price dispersion. A second approach is to introduce sticky wages. With sticky wages and flexible prices, there is no price dispersion. This means that the feedback channel described above is absent and more strongly diminishing returns steepen the Phillips curve. In a more general model with both sticky wages and prices, the presence of sticky wages will dampen the economy-wide increase in wages following a demand change. Indeed, with sufficiently sticky wages (for example, using the calibration from Erceg, Henderson, and Levin, 2000), more strongly diminishing returns steepen the Phillips curve.

The rest of this paper proceeds as follows. In Section 2, we set out the model and derive the basic result described above. In Section 3, we show that this result does not hold in a model with Rotemberg pricing or a model with Calvo-type stickiness in wages. In Section 4, we consider what happens to inflation in response to an increase in aggregate demand which is only gradually removed. The findings in Section 2 and 3 on the effect of more strongly diminishing returns on the slope of the Phillips curve remain true in these more general circumstances.

2 The Model

This section presents the model in Galí (2015, Chapter 3). There is a representative household that chooses consumption and labour supply. Firms compete under monopolistic competition and choose prices subject to price adjustment frictions à la Calvo. This section briefly presents each model block, the equilibrium condition, the Phillips curve, and the main result on how more strongly diminishing returns affect the slope of the Phillips curve.

2.1 The Household

The representative household is infinitely-lived and maximises the sum of discounted utility from consumption C and disutility from labour N

$$\max \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{2}$$

subject to

$$P_t C_t + \frac{B_t}{1+i_t} \leq B_{t-1} + W_t N_t + D_t \tag{3}$$

where P_t is the price of consumption goods, B_t is a one-period nominal bond, i_t is the nominal interest rate between period t and $t+1$, W_t is the wage, and D_t is dividend from firm ownership. $\beta \in (0,1)$ is the discount factor, σ is the inverse of the elasticity of intertemporal substitution, and ϕ is the inverse of the Frisch elasticity of labour supply. We further define

$$C_t = \left(\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where ε is the elasticity of substitution between goods and j is an index of goods. Maximizing total consumption subject to a given level of spending gives rise to the set of demand functions

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} C_t \quad \forall \quad j, \tag{5}$$

and the following aggregate price index

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{1/(1-\varepsilon)}. \tag{6}$$

2.2 The Firms

There is a continuum of firms indexed by j that compete under monopolistic competition and face pricing frictions of the Calvo type, where they can reset prices in a period with probability $1-\theta$. The firm problem is

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \theta^k \left(\frac{U_{C,t+k}}{U_{C,t}} \left(\frac{P_{j,t}}{P_{t+k}} Y_{j,t+k} - \frac{W_{t+k}}{P_{t+k}} N_{j,t+k}\right)\right) \tag{7}$$

$$Y_{j,t+k} = N_{j,t+k}^{1-\alpha} \tag{8}$$

$$Y_{j,t+k} = \left(\frac{P_{j,t+k}}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{9}$$

where W is the nominal wage, N is labour, and Y is output. Equation (8) is the production function where we assume that there are diminishing returns to labour in production, governed by the parameter $\alpha \in [0, 1)$. Equation (9) is derived by combining Equation (5) with the closed economy market clearing for goods in all markets $C_{j,t} = Y_{j,t}$.

2.3 Equilibrium

The competitive equilibrium consists of vectors of prices and quantities consistent with the household and firms solving the abovementioned problems and market clearing conditions for all markets. The goods market clearing condition is $Y_{j,t} = C_{j,t} \forall j$ (which implies that $Y_t = C_t$), and the labour market clearing is that labour supply N_t equals labour demand N_t.

2.4 The Phillips Curve

To derive the Phillips curve (the relationship between inflation and output), we start from the log-linearized solution to the firm problem

$$p_t^* = -mc + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathbb{E}_t\left\{\psi_{t+k|t}\right\} \tag{10}$$

where p_t^* is the logarithm of the optimal price, mc is the logarithm of the steady state real marginal costs, and $\psi_{t+k|t}$ is the logarithm of nominal marginal cost for a firm that last reset its price in period t. Nominal marginal costs are defined as $\Psi_{t+k|t} = \frac{W_{t+k}}{MPN_{t+k|t}}$ while the average nominal marginal costs are $\Psi_{t+k} = \frac{W_{t+k}}{MPN_{t+k}}$. Hence,

$$\psi_{t+k|t} - \psi_{t+k} = -\left(mpn_{t+k|t} - mpn_{t+k}\right) = \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) = -\frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k}).$$

Now replace for $\psi_{t+k|t}$ in equation (10)

$$p_t^* = -mc + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathbb{E}_t\left\{\psi_{t+k} - \frac{\varepsilon\alpha}{1-\alpha}\left(p_t^* - p_{t+k}\right)\right\}$$

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathbb{E}_t\left\{p_{t+k}\right\} + \frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathbb{E}_t\left\{\widetilde{mc}_{t+k}\right\} \tag{11}$$

where $\widetilde{mc}_{t+k} = \psi_{t+k} - p_{t+k} - mc$ is the deviation of log real marginal costs from its steady state value. Rewrite equation (11) into an iterative equation

$$p_t^* - p_{t-1} = \pi_t + \beta\theta\mathbb{E}_t\left\{p_{t+1}^* - p_t\right\} + \frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}(1-\beta\theta)\,\widetilde{mc}_t$$

and use the definition of aggregate inflation behavior $\left(\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)\right)$ to get

$$\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \frac{(1-\theta)(1-\beta\theta)}{\theta\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right)}\widetilde{mc}_t. \tag{12}$$

To rewrite equation (12) into the standard Philips curve, note that $\widetilde{mc}_t = \widetilde{\omega}_t - \widetilde{mpn}_t = \left(\sigma + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right)\tilde{y}_t$, where $\widetilde{\omega}_t$ is the log deviation of the real wage from its steady state and $\widetilde{mpn}_t$ is the log deviation of the average marginal product of labour from its steady state. The Phillips curve in this economy is thus

$$\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa\tilde{y}_t, \tag{13}$$

where the slope κ is given by

$$\kappa = \underbrace{\left[\sigma + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]}_{1} \underbrace{\left[\frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}\right]}_{2} \underbrace{\left[(1-\beta\theta)\frac{(1-\theta)}{\theta}\right]}_{3} \tag{14}$$

Our focus is on the role of diminishing returns for the slope of the Phillips curve, that is, on how changes in α affect the effect of y_t on π_t. We will first consider the effect on π_t of a one-off change in y_t which is immediately reversed. We will consider the effect of a persistent change in y_t on π_t in Section 4 below.

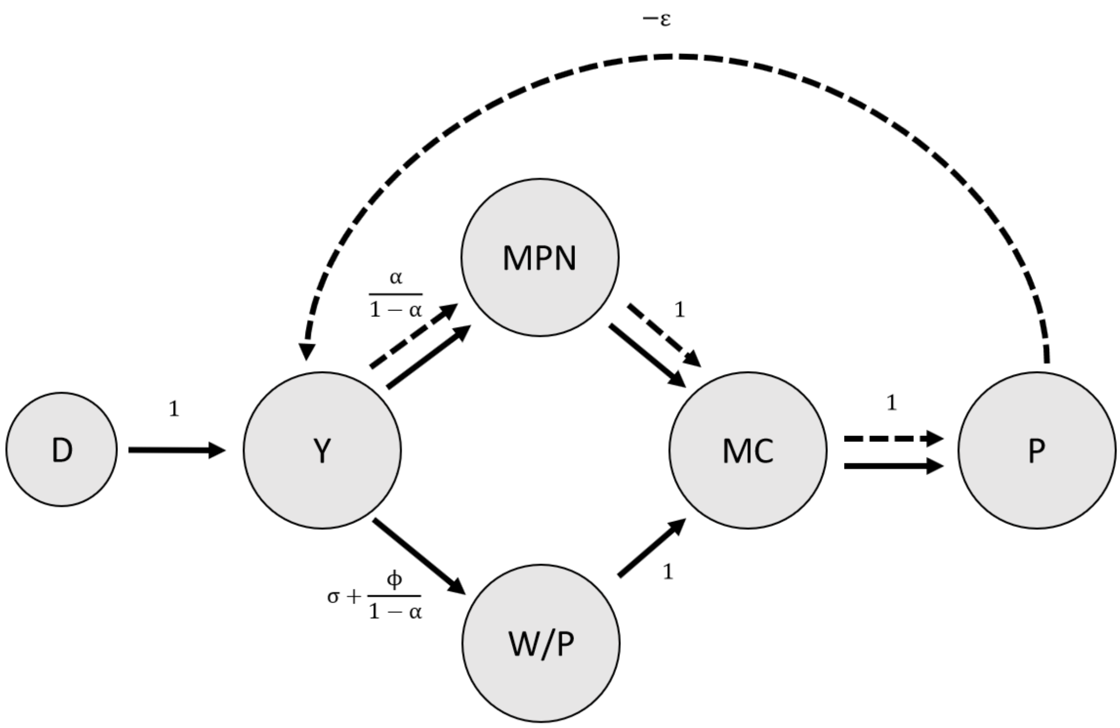
We first notice that, since the model is a forward-looking model with no persistence, a change in π_t will not affect $\mathbf{E}_t\{\pi_{t+1}\}$. This means that we do not need to consider any feedback effect of π_t on $\mathbf{E}_t\{\pi_{t+1}\}$, and thus back on π_t when considering the effect of y_t on π_t. Thus, we only need to examine the effect of how changes in α affect the effect of y_t on π_t operating through the way in which any change in α affects the size of κ in Equation (2.4).[1]

We first explain the nature of the term $\left[\frac{(1-\theta)}{\theta}(1-\beta\theta)\right]$ in equation (2.4). This term captures two channels of influence on the slope of the Phillips curve. The first part of this term reflects that only a proportion $(1-\theta)$ of firms is visited by the Calvo fairy in any period. So, when there is a change in output, and thus also the current marginal costs of all firms, the firms visited by the Calvo fairy will need to know the proportion of firms θ which the fairy has not visited. For these firms to achieve any particular change in their price level relative to the price level of all other firms, they will need to increase their prices by $1/\theta$. But of course, only a proportion $(1-\theta)$ of firms will, like them, have been visited by the Calvo fairy. So the overall effect of their action on the general price level will be $(1-\theta)/\theta$. The second part of this term reflects that because firms are forward-looking, they care not only about what is happening in the current period but also about what will happen in the future. In particular, they care about the stickiness of prices θ, which determines how quickly they can reset their price in the future, and on β which shows the extent to which the future is discounted. The more forward-looking a firm is (i.e., the closer β is to unity) or the more sticky prices are (i.e., the closer θ is to unity), the less weight the firms will put on current marginal costs (and thus current output), in the optimal price-setting decision. The important thing to notice here is that although this term of course influences the slope of the Phillips curve, it is not, itself, influenced by the strength of diminishing returns.

We now consider the other two terms in Equation (2.4). These terms show the extent to which a firm will be influenced when settings its prices in period t by what it expects to happen during that period, including by what it expects to happen to the prices of other firms during that period.

There are three channels from output changes to prices, two direct and one indirect. First, to increase output, firms need more labour supply from the household. Firms have to increase the real wage to get the household to supply more labour. With higher real wages, marginal costs

[1] Thus would change if inflation expectations were to be partly backwards-looking.



Notes: The figure depicts the channels through which a demand shock D affects prices P via its effects on output Y, the marginal product of labour MPN, the real wage W/P, and marginal costs MC.

Figure 1: The effect of a demand shock (D) on prices (P) in the New Keynesian model

also increase, thus raising firms' desired prices. This channel is described by the term $\sigma + \frac{\phi}{1-\alpha}$ in Equation (2.4) and is depicted by the solid line going from Y to W/P in Figure 1.

Second, because firms face diminishing returns, higher output and thus more labour supply result in a lower marginal product of labour. A lower marginal product of labour implies that firms' marginal costs increase, raising firms' desired price. This channel is described by $\frac{\alpha}{1-\alpha}$ in Equation (2.4), and is depicted as the solid line going from Y to MPN in Figure 1.

Third, there is an indirect feedback channel from firms' price changes to firms' marginal costs. When firms who adjust prices (the "adjusters") raise the price, but the other firms (the "non-adjusters") do not, the demand for adjusters' goods fall. Lower demand implies lower production, and because adjusters face diminishing returns in production, lower production implies a higher marginal product of labour. This higher marginal product of labour reduces marginal costs, weakening firms' initial need to raise prices. This channel is described by $\frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}$ in Equation (2.4) and depicted as the dashed lines from P to Y to MPN to MC in Figure 1.

Our investigation is concerned with the role of more strongly diminishing returns in affecting the slope of the Phillips curve. More strongly diminishing returns affect both the strength of the direct channels from output to marginal costs and the indirect feedback effect from higher (relative) prices to marginal costs via lower demand and a higher marginal product of labour. Because more strongly diminishing returns have these opposing effects on the slope of the Phillips curve, the sign of the total effect is ambiguous.

However, we can directly compute how a change in α affects κ to understand what determines the sign of this total effect

$$\frac{\partial \kappa}{\partial \alpha} = \frac{\left(1 + \phi - \varepsilon\left(\sigma + \phi\right)\right)}{(1 + (\varepsilon - 1)\alpha)^2} \left[(1 - \beta\theta)\frac{1 - \theta}{\theta}\right]. \tag{15}$$

Equation (15) states that the effect of more strongly diminishing returns (an increase in α) on the slope of the Phillips curve (κ) is negative if $\varepsilon(\sigma + \phi) - (1 + \phi) > 0$.

The intuition can be seen in Figure 1. More strongly diminishing returns strengthen both direct effects but also amplify the feedback channel, which dampens the total effect. If σ and ϕ are both large, so that the elasticity of intertemporal substitution and the Frisch elasticity are both small, the real wage is more responsive to output changes and the total effect will be negative such that more strongly diminishing returns will flatten the Phillips curve.

To see the importance of the real wage response to output, assume that real wages do not respond to output. In our model, this is the same as setting $\sigma = \phi = 0$, which immediately implies that the more strongly diminishing returns will unambiguously steepen the Phillips curve. As we strengthen the real wage response to output (e.g., increase σ), the sign eventually flips and more strongly diminishing returns flatten the Phillips curve. Hence, the more the direct effect is dominated by how output changes affect real wages, the more strongly diminishing returns primarily affect the slope of the Phillips curve by dampening this wage effect.

We can thus formulate the condition loosely as follows: if the economy-wide increase in the wage is sufficiently large, more strongly diminishing returns imply that firms would raise prices by less than they otherwise would have because more strongly diminishing returns imply that raising their relative price increases their marginal product of labour more, reducing their marginal costs more and thus their need to raise prices.

Note that the condition $\varepsilon(\sigma + \phi) - (1 + \phi) > 0$ is satisfied for all reasonable calibrations of the New Keynesian model with Calvo pricing frictions. To see this, note that the condition becomes $(\varepsilon - 1)\phi > 1$ if we assume σ is zero. Recall that ϕ is the inverse of the Frisch elasticity and ε is the elasticity of substitution between goods. Chetty, Guren, Manoli, and Weber (2011) review estimates of the Frisch elasticity, arguing that a macro Frisch elasticity above one is inconsistent with the micro evidence, implying that ϕ (its inverse) is greater than 1. Moreover, in the model, ε governs the average markup, defined as $\varepsilon/(\varepsilon - 1)$. The average markup is typically calibrated to match the profit share of income in the national accounts, implying values of ε in the range of six to ten (markup of 1.10-1.20 or a profit share of 9-17%). Under these reasonable restrictions, the condition is always satisfied, and more strongly diminishing returns flatten the Phillips curve. Of course, if σ is also large, then that will ensure that the condition holds in even more general cases.

A natural question is to ask whether the open economy extension of the model in Gali and Monacelli (2005) impacts our results. The Phillips curve in the open economy model is

$$\pi_t^h = \beta \mathbf{E}_t\{\pi_{t+1}^h\} + \kappa^{\text{open}} \tilde{y}_t \tag{16}$$

where π^h is price growth on home goods and

$$\kappa^{\text{open}} = \left[\sigma^{\text{open}} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\left[\frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}\right]\left[(1-\beta\theta)\frac{(1-\theta)}{\theta}\right].$$

The only difference between κ and κ^{open} is that the σ in the expression now depends on the home bias in consumption, the elasticity of substitution between home and foreign goods, and the elasticity of substitution between goods in the foreign sector.[2] However, because σ was irrelevant in the argument above (we set it to 0), extending the model with openness does not alter the *qualitative result* that more strongly diminishing returns flatten the Phillips curve in all reasonable calibrations of the model. However, because it affects the size of σ, it will affect the slope of the Phillips curve itself and the extent to which more strongly diminishing returns flatten the Phillips curve.

3 Alternative Models

There are two ways to avoid the seemingly unintuitive result that more strongly diminishing returns flatten the Phillips curve. First, one may use a price stickiness framework that does not feature the feedback channel from relative prices to marginal costs or causes it to become much weaker. This is the same as removing the dashed arrows in Figure 1. Second, one may dampen the effect of output changes on real wages, for example, by introducing sticky nominal wages. This is the same as dampening the solid arrow from Y to W/P in Figure 1. This section presents examples of these two approaches.

3.1 Rotemberg Pricing

The first way to avoid the result that more strongly diminishing returns flatten the Phillips curve is to use a Rotemberg pricing model with convex adjustment costs (Rotemberg, 1982) rather than a Calvo pricing friction. We assume firms now face quadratic price adjustment costs of the form

$$\frac{\Theta}{2}\left(\frac{P_{i,t+k}}{P_{i,t+k-1}} - 1\right)^2 Y_{t+k} \tag{17}$$

[2]See Gali and Monacelli (2005) or Galí (2015, Chapter 8) for derivations.

where Θ is a parameter determining the adjustment costs. The log-linearized Phillips curve with Rotemberg pricing (around a zero inflation steady state) has the same form as before (the model solution is derived in Appendix A)

$$\pi_t = \beta \mathbf{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t.$$

However, the expression for the slope of the Phillips curve differs from the Calvo model and is now given by

$$\kappa = \left[\sigma + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\left[\frac{\varepsilon - 1}{\Theta}\right]. \tag{18}$$

In this Rotemberg case, there is no longer a feedback channel from relative price movements via the marginal product of labour to individual firms' marginal costs. In expression (18), this is evident as the absence of the term $\left[\frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}\right]$. The intuitive reason is that because all firms choose the same price with Rotemberg pricing, there is no price dispersion and thus no feedback channel from how a change in an individual firm's price affects its demand and marginal costs. Assuming Rotemberg pricing thus removes the feedback channel.

The immediate result is that more strongly diminishing returns unambiguously *steepen* the Phillips curve. This can be seen by taking the derivative of κ in (18) with respect to α. The intuition is that more strongly diminishing returns steepen the Phillips curve because more strongly diminishing returns strengthen the channels from output to marginal costs.

3.2 Wage Stickiness

The second way to avoid the result that more strongly diminishing returns flatten the Phillips curve is to introduce sticky wages. To illustrate how sticky wages might affect our results, we use the framework deployed by Erceg et al. (2000).[3] Following the approach in that paper, we assume there exists a continuum of worker types, all of which are used by each firm in the same proportion, each of which has its own wage-setting union. Also - again following Erceg et al. - we assume that only some of the unions are visited by the Calvo fairy in each period. But, to clarify the discussion, unlike in Erceg et al., we suppose there is only wage stickiness, not price stickiness. Because all firms employ the same kinds of labour in the same proportions, all firms will face the same overall wage increase when output increases.[4] And therefore, because there is no price stickiness, all firms will increase their prices in the same way.

The analysis required in this case, in which wages are sticky but prices are flexible, is complex. But the kind of complexity involved parallels the kind of complexity we encountered when working through the main case considered earlier in the paper - the Calvo-model case - in which wages are flexible, but prices are sticky.

[3]This approach is set out carefully in Galí (2015, Chapter 6).

[4]See Galí (2015, p. 165).

The analysis proceeds as follows. Assume that a union setting wages can reset the wage of its workers in any period with probability $(1-\theta_w)$. Then any union solves the following problem

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \theta_w^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{u,t+k} - \frac{N_{u,t+k}^{1+\phi}}{1+\phi} \right) \tag{19}$$

$$N_{u,t+k} = \left(\frac{W_{u,t+k}}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}. \tag{20}$$

The log-linearized wage Phillips curve is (see calculation in Appendix B)

$$\pi_t^w = \beta \mathbb{E}_t\{\pi_{t+1}^w\} + \zeta_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{21}$$

with

$$\zeta_w = \left[\sigma + \frac{\phi}{1-\alpha}\right]\lambda_w, \quad \lambda_w = \left[\frac{1}{1+\varepsilon_w \phi}\right]\left[\frac{1-\theta_w}{\theta_w}\right][1-\beta\theta_w]$$

where $\tilde{\omega}_t$ is the deviation of the real wage from its steady state determined by

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t. \tag{22}$$

With flexible prices, inflation is equal to a markup over marginal costs, implying that

$$\pi_t = \pi_t^w + \frac{\alpha}{1-\alpha}(\tilde{y}_t - \tilde{y}_{t-1}). \tag{23}$$

An immediate implication is that the mechanism, present in the Calvo model in Section 2, through which more strongly diminishing returns might cause the Phillips curve to flatten, is entirely absent here. As already noted, all firms face the same overall wage increase, so because prices are flexible, they will increase their prices by the same amount. There is no longer a feedback channel in which a firm, when adjusting its price, has to be concerned about how any price increase will increase the price of its product relative to the price of its competitors who do not adjust their prices. It was a feedback channel of this kind that gave rise to the possibility that more strongly diminishing returns might flatten the Phillips curve. Therefore, in this case with sticky wages, more strongly diminishing returns unambiguously *steepen* the Phillips curve.[5]

[5]Things would be different if each firm only employed its own kind of labour and if the Calvo wage-fairy only visited unions representing some kinds of labour, and therefore only affected the wage of some firms. Even although there was price flexibility, the prices set by the firms in which wages had increased would be higher than the prices set by the other firms. And so there would once again be a feedback channel like that in our previous analysis. The unions, when adjusting their wages, would need to be concerned about how the resulting increase in the price set by the firms whose workers they represented would increase the price of that firm's product relative to the price of the goods produced by the firms which had not been visited by the Calvo wage-fairy, thereby reducing the demand for the firm's product, and so the demand for their labour. This channel would influence the effect of more strongly diminishing returns on the slope of the Phillips curve. We have not studied this case in detail. However, we would expect this channel to have the

Wage and price stickiness. In addition, we examine the case considered in Erceg et al. (2000) in which there is price and wage stickiness where we replace (23) with the following price Phillips curve

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \zeta_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{24}$$

with $\zeta_p = \frac{\alpha}{1-\alpha}\lambda_p$ and $\lambda_p = \frac{1-\theta}{\theta}(1-\beta\theta)\frac{1}{1+\frac{\varepsilon\alpha}{1-\alpha}}$ (see Galí, 2015, Chapter 6). In this model, it is more complicated to determine how more diminishing returns affect the slope of the Phillips curve because the total effect depends on how the rest of the economy responds. However, we can gauge where to look by taking the derivative of (24) with respect to the output gap

$$\frac{d\pi_t}{d\tilde{y}_t} = \zeta_p + \lambda_p \frac{d\tilde{\omega}_t}{d\tilde{y}_t} + \beta \frac{d\mathbb{E}_t\{\pi_{t+1}\}}{d\tilde{y}_t}.$$

The two objects that are comparable with κ in the sticky price model above are ζ_p and $\lambda_p \frac{d\omega_t}{d\tilde{y}_t}$. Next, note that $\partial\zeta_p/\partial\alpha > 0$ and $\partial\lambda_p/\partial\alpha < 0$. Informally, more diminishing returns steepen the Phillips curve if $\frac{d\tilde{\omega}_t}{d\tilde{y}_t}$ is small enough or negative and not too responsive to changes in α. This can be achieved if wages are sufficiently sticky relative to prices.

We can therefore conclude that in this model, a sufficient degree of wage stickiness will mean that - if all other parameters of the Calvo price-stickiness model remain as in the case considered earlier in the paper - it will *not* be the case that a more strongly diminishing returns will flatten the Phillips curve. The reason for this is as follows. In this model, the feedback channel discussed previously would return. That is because each firm that adjusts its price would need to be concerned about the way in which its price increase would increase the price of its product relative to its non-adjusting competitors. And each of these adjusting firms will face a generalised wage increase, which might lead to a situation in which more diminishing returns might cause a flattening of the Phillips curve. But with wage stickiness, the generalised wage increase faced by the adjusting firms will be (much) smaller.

4 Simulation Results

In this section, we illustrate how more diminishing returns affect the slope of the Phillips curve in a dynamic setup in which the shock to output is non-transitory. We do this for three models: the model with Calvo pricing frictions, the model with Rotemberg pricing frictions, and the model with sticky wages (with and without sticky prices). In each case, we simulate the inflation response to a persistent demand shock. This is simple to do because, in each case, the following will be true as a result of the output shock

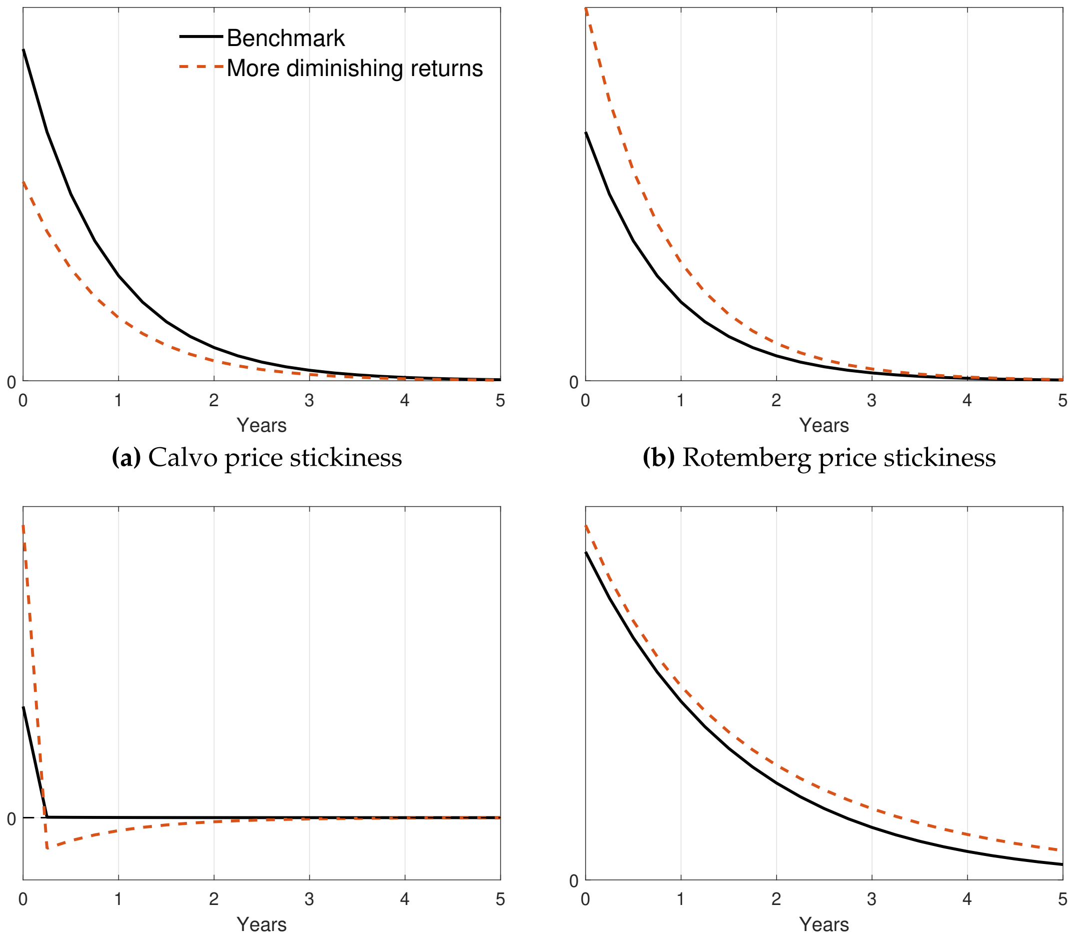
$$\pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1} = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t y_{t+k} \tag{25}$$

same direction of effect on inflation as the price stickiness identified in the Calvo model.

and we can simply plug in y_{t+k} from our assumptions about the persistence of the output shock. In all three models, we use the following setup

$$\tilde{y}_t = \mathbf{E}_t\{\tilde{y}_t\} - \frac{1}{\sigma}(r_t - r) + d_t,$$
$$d_t = \rho d_{t-1} + \varepsilon_t^d,$$

where r_t is the real interest rate, r is the steady state real interest rate, d_t is a demand innovation that follows an AR(1) with persistence ρ, and ε_t^d is a demand shock. We assume that the central bank always sets $r_t = r$ so that the demand shock affects the output gap one for one.



(a) Calvo price stickiness

(b) Rotemberg price stickiness

(c) Sticky wages

(d) Sticky wages and prices

Figure 2: The inflation response to a demand shock.

We follow Galí (2015) when we calibrate the models. We assume the following calibration across all models: $\sigma = 1$, $\phi = 5$, $\beta = 0.99$, $\varepsilon = 9$, $\alpha = 0.25$, and $\rho = 0.75$. In the specific models, we assume $\theta = 0.75$ in the Calvo model, $\Theta = 372.8155$ (same slope of the Phillips curve as in the

Calvo model) in the Rotemberg model, and $\theta_w = 0.75$ and $\varepsilon_w = 4.5$ in the sticky wage model and the sticky wage and price model (similar to Galí, 2015, Chapter 6).

Figure 2 displays the four models' inflation responses to a demand shock in the benchmark calibrations ($\alpha = 0.25$) and calibrations with more strongly diminishing returns ($\alpha = 0.50$). In the Calvo model, more strongly diminishing returns weaken the inflation response. In contrast, more strongly diminishing returns strengthen the inflation response in the Rotemberg, the sticky wage, and the sticky wage and price models. The three alternative models are therefore consistent with our initial intuition that more strongly diminishing returns *steepen* the Phillips curve because it implies that an increase in demand will reduce firms' marginal product of labour by more, thus raising their marginal costs and inducing them to increase prices by more.[6]

Among the alternative models, the result that more diminishing returns steepen the Phillips curve holds for all calibrations only for the Rotemberg pricing and the sticky wage (and flexible price) models. For the model with sticky prices and wages, the impact of more diminishing returns on the slope of the Phillips curve depends on the specific calibration. In the calibration we use in Figure 2(d), prices and wages are equally sticky ($\theta = \theta^w = 0.75$). If we instead assume that prices are more sticky than wages, we eventually return to the case in Figure 2(a), where more diminishing returns flatten the Phillips curve.

5 Conclusion

This paper asks how more strongly diminishing returns affect the slope of the Phillips curve in New Keynesian models. We first show that more strongly diminishing returns flatten the Phillips curve in the New Keynesian model with Calvo pricing frictions for all reasonable calibrations. This is because there is a feedback channel in the Calvo model in which adjusters' relative prices affect their demand, the marginal product of labour, and marginal costs. Because, for these calibrations, real wages respond to output by a significant amount, the feedback channel dominates, and more strongly diminishing returns flatten the Phillips curve.

We present two ways of getting the alternative and perhaps more intuitive result that more strongly diminishing returns steepen the Phillips curve. First, one can assume Rotemberg pricing, which removes the feedback channel discussed above. Second, one can assume sticky wages, which dampens the importance of real wage adjustments for average marginal costs, and thus reduces the importance of the feedback channel. In standard calibrations of these alternative models, more strongly diminishing returns *steepen* the Phillips curve.

Our results are important for an understanding of how supply constraints affect the inflation

[6]The intuition for Figure 2(c) (the sticky-wage case) might initially seem surprising. In that case, prices jump on impact so much that they overshoot and then have to revert back. They do this because, although prices are flexible, wages are sticky. Hence, prices move mainly in response to a lower marginal product of labor (due to higher output), but revert once output returns to its steady state. Conversely, in the sticky-price case with flexible wages, wages also have a similar pattern: wages jump on impact and then revert.

process. It is clearly necessary to understand the effects on that process of exogenously imposed supply constraints, such as Covid-related lockdowns, or of endogenous supply constraints, such as hysteresis effects following on from recessions. Our paper shows that a researcher's choice of pricing frictions, when formulating the inflationary model, will have a first-order impact on the researcher's understanding of these effects. The results obtained in our paper are thus of considerable importance.

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Appendix

A The Rotemberg Pricing Model

The Rotemberg price-setting problem is

$$\max_{P_{i,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \frac{U_{C,t+k}}{U_{C,t}} \left\{ \frac{P_{i,t+k}}{P_{t+k}} Y_{i,t+k} - \frac{W_{t+k}}{P_{t+k}} N_{i,t+k} - \frac{\Theta}{2} \left(\frac{P_{i,t+k}}{P_{i,t+k-1}} - 1 \right)^2 Y_{t+k} \right\} \tag{A.1}$$

subject to

$$Y_{i,t+k} = N_{i,t+k}^{1-\alpha}, \tag{A.2}$$

$$Y_{i,t+k} = \left(\frac{P_{i,t+k}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}, \tag{A.3}$$

where Θ is the adjustment cost and U_C is marginal utility. The first-order condition is

$$1 - \Theta(\Pi_t - 1)\Pi_t + \Theta\beta \frac{U_{C,t+1}}{U_{C,t}} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} = (1 - MC_t)\varepsilon \tag{A.4}$$

where $\Pi_t = P_t/P_{t-1}$ and $MC_t = \frac{W_t/P_t}{MPN_t}$ is the real marginal costs. The log-linearized solution around $\Pi = 1$ is

$$\pi_t = \beta \mathbf{E}_t \pi_{t+1} + \frac{\varepsilon - 1}{\Theta} \widetilde{mc}_t \tag{A.5}$$

where

$$\widetilde{mc}_t = \left(\sigma + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha} \right) \tilde{y}_t.$$

We thus have the Phillips curve

$$\pi_t = \beta \mathbf{E}_t \pi_{t+1} + \left(\sigma + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha} \right) \left(\frac{\varepsilon - 1}{\Theta} \right) \tilde{y}_t. \tag{A.6}$$

B The Wage Phillips Curve

This appendix shows how we derive the wage Phillips curve (21).

The first-order conditions of the Union problem in Section 3.2 is

$$\pi_t^w = \beta \mathbb{E}_t\{\pi_{t+1}^w\} - \lambda_w \tilde{\mu}_t^w \tag{B.1}$$

where $\tilde{\mu}_t^w$ is the log deviation of the average wage markup from its steady state and

$$\lambda_w = \left[\frac{1}{1+\varepsilon_w \phi}\right]\left[\frac{1-\theta_w}{\theta_w}\right][1-\beta\theta_w].$$

The log wage markup is defined as

$$\mu_t^w = \omega_t - mrs_t,$$

which implies that

$$\tilde{\mu}_t^w = \mu_t^w - \mu^w = \tilde{\omega}_t - \widetilde{mrs}_t = \tilde{\omega}_t - \left(\sigma + \frac{\phi}{1-\alpha}\right)\tilde{y}_t$$

where $\widetilde{mrs}_t$ is the log deviation of the marginal rate of substitution between consumption and leisure and $\tilde{\omega}_t$ is the log deviation of the real wage from its steady state. The wage Phillips curve is then

$$\pi_t^w = \beta \mathbb{E}_t\{\pi_{t+1}^w\} + \zeta_w \tilde{y}_t - \lambda_w \tilde{\omega}_t$$

where

$$\zeta_w = \left[\sigma + \frac{\phi}{1-\alpha}\right]\lambda_w.$$