

Monetary Transmission with Income Risk*

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March 2022

Abstract

In periods of distress, observed and perceived income risk tends to rise. Does this heightened income risk affect monetary transmission? This paper first shows that in partial equilibrium, heightened income risk *dampens* the substitution effect of interest rate changes but *amplifies* the indirect income effect of wage changes. The effects are sizable in partial equilibrium. An increase in income risk consistent with heightened risk during recessions affects interest rate and wage responses by around 1/3. However, since income risk *dampens* the effects of interest rate changes but *amplifies* the effects of wage changes, its effect is weaker in general equilibrium, dampening monetary transmissions to consumption by around 11%.

JEL: D14, D52, E52

Keywords: Income Risk, Monetary Policy

*Department of Economics, University of Oslo; e-mail: martin.b.holm@outlook.com; web: <https://www.martinbholm.com/>. I thank Pontus Rendahl (the editor) and two anonymous referees for excellent comments and suggestions. I would also like to thank SeHyouun Ahn, Simon Galle, Even Hvinden, Ragnar Juelsrud, Benjamin Moll, Gisle J. Natvik, Plamen Nenov, Morten O. Ravn, Kjetil Storesletten, Tommy Sveen, and various conference and seminar participants for insightful comments and discussions.

1 Introduction

This paper asks how income risk affects monetary transmission. This question is important because perceived and observed income risk tends to rise in times of distress.¹ If income risk dampens monetary transmission, monetary policy may be a weaker tool when it is needed. Hence, business cycle variation in income risk may help explain why monetary policy seems to be less potent in recessions than in expansions (Tenreyro and Thwaites, 2016).

A change in income risk affects monetary transmission through two channels. First, it affects the behavior of individuals by changing their optimal responses to shocks. This channel operates regardless of whether the increase in risk is actual or perceived. Second, higher income risk affects the distribution of shocks and state variables by, for example, affecting the level of inequality and thus how the economy responds to monetary policy. This paper focuses exclusively on the first channel to isolate the impact of higher perceived income risk on monetary transmission.

To understand how income risk affects monetary transmission, Section 2 first investigates theoretically how income risk affects non-durable consumption responses to interest rate and wage changes for an *individual household*. The starting point is a consumption-saving problem in an incomplete-markets setting with idiosyncratic income risk. The household consumes, saves, and borrows but lacks insurance against the idiosyncratic income risk it faces. In this framework, Section 2 explores how a household responds to changes in the interest rate and wage, and how these responses depend on the level of income risk. The main result is that income risk *dampens* the consumption response to interest rate changes but *amplifies* the consumption response to wage changes.

Section 3 and 4 next assess whether income risk may have quantitatively relevant effects on monetary transmission. The first step is to evaluate whether each channel derived theoretically may be quantitatively relevant in a partial equilibrium setting. Section 3 simulates individual consumption responses to interest rate and wage change in two cases: low-income risk and high-income risk. The high-income risk case is computed as the responses to interest rate and wage changes with the policy functions consistent with higher income risk.² The difference between the two cases reveals the *isolated* impact of higher perceived income risk on the consumption responses to interest rate and wage changes, holding everything else constant. The second main contribution lies here. Consistent with the theoretical predictions, an increase in income risk of a similar magnitude as the change in risk from expansion to recession from Storesletten et al. (2004) *dampens* a household's sensitivity to short-run interest rate changes by around 1/3 but *amplifies*

¹See, e.g., Storesletten, Telmer, and Yaron (2004) (higher standard deviation of income in recessions), McKay (2017) (more long-term earnings losses in recessions) and Guvenen, Ozkan, and Song (2014) (more left-skewness in recessions).

²The impulse responses in the high-risk case are defined as the difference between the responses to two shocks (higher perceived income risk and monetary policy) minus the response to higher perceived income risk in isolation. Note that this has nothing to do with countercyclical or procyclical income risk. It is merely a way to ensure that the two cases' initial conditions are the same. The responses are similar (with opposite sign) if one considers the case with lower income risk.

the sensitivity to wage changes by around 1/3. Hence, income risk has potentially sizable effects on interest rate and wage sensitivity. However, the two effects may partially cancel out because monetary policy affects consumption through interest rates and wages.

The main drawback of the theoretical results in Section 2 and the quantitative exploration in Section 3 is that they are limited to a partial equilibrium setting. Exploring this question in general equilibrium is important for two reasons. First, income risk affects several monetary transmission channels as explained above. Second, prices may move endogenously to changes in perceived income risk and hence the sum of partial equilibrium effects is not similar to the total effect in general equilibrium. Section 4 presents a one-asset heterogeneous agent New-Keynesian model to explore the role of income risk for monetary transmission in a general equilibrium setting. Using this model, one can compare how the economy responds to monetary policy in two cases: constant income risk and along the transition path of a simultaneous perceived increase in income risk. The last main result lies here: while income risk dampens monetary transmission also in a general equilibrium setting, the quantitative effect is small. A change in the level of income risk from its value in the 1970s to today's value dampens monetary transmission to aggregate consumption by around 11%. Hence, this paper's main message is that while income risk may significantly affect individuals' responses to interest rate and wage changes, its impact on aggregate monetary transmission is small.

Literature. The theoretical results belong to a growing literature on theoretical results for incomplete markets economies. Common to all these papers is that they apply sufficient restrictions to derive theoretical results. For example, [Bilbiie \(2008, 2018, 2020\)](#) assume limited asset market participation, [Acharya and Dogra \(2020\)](#) and [Acharya, Challe, and Dogra \(2020\)](#) assume a specific utility function and form of risk, and [Werning \(2015\)](#) assumes specific equilibrium properties.³ The main contribution of this paper is that this paper studies the role of high perceived income risk in isolation and focuses on the channels that operate for individual households, while the papers above study the effects of the *cyclical* of income risk in general equilibrium. Furthermore, this paper investigates the problem *quantitatively* to see whether risk may play a major role in explaining variation in monetary transmission.

Although the results in this paper imply that the level of income risk does not significantly affect aggregate monetary transmission, it does not imply that market incompleteness is irrelevant for monetary policy. In particular, this paper finds that income risk changes the relative importance of monetary transmission channels. Consistent with [Kaplan, Moll, and Violante \(2018\)](#) and [Kaplan and Violante \(2018\)](#),⁴ a higher income risk weakens the substitution effect from interest rate changes

³Similarly, [Broer, Harbo Hansen, Krusell, and Öberg \(2020\)](#) study the importance of profits for monetary transmission in a model with no liquidity.

⁴Similarly, there is growing literature on monetary policy models with incomplete markets, including [McKay, Nakamura, and Steinsson \(2016\)](#); [Auclert \(2019\)](#); [Lueticke \(2021\)](#); [Guerrieri and Lorenzoni \(2017\)](#) that highlights essential mechanisms that may affect monetary transmission.

but strengthens the indirect income effects. The results in this paper thus complement this literature by analyzing the isolated impact of income risk on monetary transmission.

It is essential to keep in mind that income *risk* and income *inequality* are not the same things. There is an emerging literature that focuses on the quantitative effects of increased income *inequality* on the economy. For example, [Werning \(2015\)](#) analyzes the impact of inequality on aggregate demand policies and [Auclert and Rognlie \(2018\)](#) focus on the long-run impact of inequality on the economy. Similarly, [Straub \(2017\)](#) shows that consumption is concave in permanent income, implying that an increase in permanent income inequality results in a decline in interest rates and an increase in the wealth-to-income ratio (see also [Mian, Straub, and Sufi, 2021](#)). This paper differs from this literature by analyzing the isolated impact of income *risk*, not the effects of income *inequality*.

Roadmap. The rest of this paper is structured as follows. Section 2 presents the main intuition in a two-period model. Section 3 and 4 then present the results from quantitative simulations of a partial equilibrium model and a general equilibrium heterogeneous agent New-Keynesian model, respectively. Section 5 concludes.

2 A Simplified Household Model

This section aims to provide intuition for how income risk affects monetary transmission. To that end, this section analyzes how income risk affects a household's sensitivity to interest rate and wage changes in a two-period model. After providing this intuition, more general settings are analyzed in Sections 3 and 4.

2.1 The Model

The household lives for two periods and maximizes its discounted expected utility flow from consumption c

$$\max_{\{c_1, c_2\}} u(c_1) + \beta u(c_2) \tag{1}$$

subject to

$$c_1 + \frac{a}{1+r} = w \tag{2}$$

$$c_2 = a + 1 + \zeta \tag{3}$$

where $\beta \in (0, 1]$, a is a risk-free bond, w is the wage in period 1 (relative to the expected wage in period 2), r is the interest rate, and ζ is a mean-zero spread with variance σ^2 . The utility function u satisfies $u' > 0$, $u'' < 0$, and $u''' > 0$. The Euler-equation describing the optimal consumption path

is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'(c_2)\}. \quad (4)$$

2.2 Auxiliary Results

The following three paragraphs first show how consumption is affected by changes in the interest rate, the wage, and income risk, respectively before the role of income risk is explored in the next subsection.

Consumption response to interest rate changes. Lemma 1 presents how current consumption responds to a change in the interest rate.

Lemma 1. *Assume that $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$, then the response of current consumption to a change in the interest rate is*

$$\frac{dc_1}{dr} = \underbrace{\frac{\mathbb{E}\{u'(c_2)\}}{u''(c_1) + \mathbb{E}\{u''(c_2)\}}}_{\text{substitution effect}} + \underbrace{\frac{\mathbb{E}\{u''(c_2)\}[w - c_1]}{u''(c_1) + \mathbb{E}\{u''(c_2)\}}}_{\text{income effect} = 0} = \frac{1}{2} \frac{u'(c)}{u''(c)} = - \underbrace{\text{MPC}}_{=\frac{1}{2}} \cdot \underbrace{\text{EIS}}_{=-\frac{u'(c)}{u''(c)}} \cdot c \quad (5)$$

where $c = c_1 = \mathbb{E}c_2 = 1$.

The interest rate change affects current consumption through the substitution and income effect. Under the assumptions that $\beta = (1+r) = 1$, $\sigma^2 \rightarrow 0$, and $w = 1$, the original plan is to have consumption equal to the wage in both periods such that there is no income effect. Interest rate changes thus affect consumption only through the substitution effect. Since $u' > 0$ and $u'' < 0$, the substitution effect is negative: the household reduces current consumption in response to a higher interest rate because the relative price of current consumption is higher (relative to future consumption). The last term also clarifies how the strength of the response of current consumption to interest rate changes depends on (i) the marginal propensity to consume (MPC) (which is 1/2 in this two-period model) and (ii) the elasticity of intertemporal substitution (EIS).

Consumption response to wage changes. Lemma 2 presents how current consumption responds to a change in the current wage.

Lemma 2. *Assume that $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$, then the response of current consumption to a change in the current wage is*

$$\frac{dc_1}{dw} = \frac{\mathbb{E}\{u''(c_2)\}}{u''(c_1) + \mathbb{E}\{u''(c_2)\}} = \frac{1}{2} = \text{MPC}. \quad (6)$$

Current consumption increases in response to a higher current wage. Since $\beta = (1+r) = 1$ and there is no income risk, the household will smooth consumption across the two time periods. The

marginal response depends on the marginal propensity to consume (MPC), which is 1/2 in this two-period model.

Consumption response to income risk changes. Lemma 2 presents how current consumption responds to a change in income risk.

Lemma 3. *Assume that $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$, then the response of current consumption to a change in income risk is*

$$\frac{dc_1}{d\sigma^2} = \frac{1}{2} \frac{u'''(\mathbb{E}c_2)}{u''(c_1) + u''(\mathbb{E}c_2)} = \frac{1}{4} \frac{u'''(c)}{u''(c)} = -\frac{1}{4} \cdot \underbrace{\text{Absolute Prudence}}_{=-\frac{u'''(c)}{u''(c)}} \quad (7)$$

where $c = c_1 = \mathbb{E}c_2 = 1$.

Lemma 3 states that if the household is prudent, which is satisfied if $\frac{u'''(c)}{u''(c)} < 0$ (Kimball, 1990), it responds to an increase in income risk by reducing current consumption for precautionary reasons. Intuitively, the effect depends on $u''' > 0$, which ensures that the marginal utility of consumption is convex. With convex marginal utility, the household wants to transfer resources to periods where it expects variation in marginal utility (period 2), thus reducing consumption in period 1.

2.3 Does Income Risk Matter?

The previous section shows how current consumption responds to interest rate, wage, and income risk changes. This section shows how these responses depend on the level of income risk.

Income risk and interest rate sensitivity. Proposition 1 presents how the current consumption response to interest rate changes depends on income risk.

Proposition 1. *Assume that $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$, then the effect of income risk on how households respond to interest rate changes is*

$$\frac{d^2c_1}{drd\sigma^2} \approx -\frac{1}{8} \frac{u''''(c)u'(c)}{u''(c)u''(c)} \quad (8)$$

where $c = c_1 = \mathbb{E}c_2 = 1$. Equation (8) is positive if the utility function has the property decreasing absolute prudence $\frac{d(-\frac{u''''}{u''})}{dc} < 0$.

The marginal consumption response to interest rate changes is

$$\frac{\mathbb{E}\{u'(c_2)\} + \mathbb{E}\{u''(c_2)\}[w - c_1]}{u''(c_1) + \mathbb{E}\{u''(c_2)\}}.$$

Income risk affects the marginal consumption response to interest rate changes through several channels. First, since higher income risk induces households to save for precautionary reasons,

it reduces the expected marginal utility from consumption in period two due to the convexity of marginal utility and thus strengthens the substitution effect. However, it also strengthens the income effect because households save more in period 1. These two effects cancel out exactly. The only remaining effect is that higher income risk reduces $\mathbb{E}\{u''(c_2)\}$ if it is concave ($u'''' < 0$) by Jensen's inequality. This channel weakens the current consumption response to interest rate changes.

Corollary 1. *Assume that $u'(c) = c^{-\gamma}$ with $\gamma > 0$, then (8) becomes*

$$\frac{d^2 c_1}{dr d\sigma^2} \approx \frac{1}{8}(1 + \gamma)(2 + \gamma)c^{-1} > 0. \quad (9)$$

Proposition 1 further shows that (8) is positive if the utility function satisfies decreasing absolute prudence. Indeed, decreasing absolute prudence is not strictly necessary, only that the utility function has the property temperance ($u'''' < 0$). Since the power utility function has the property decreasing absolute prudence, it follows in Corollary 1 that the marginal effect of income risk on the current consumption response to interest rate changes is positive. Since the substitution effect of interest rate is initially negative, it follows that higher income risk *weakens* the substitution effect from interest rate changes.

Income risk and wage sensitivity. Proposition 2 presents how the current consumption response to wage changes depends on income risk.

Proposition 2. *Assume that $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$, then the effect of income risk on the consumption response to short-run wage changes is*

$$\frac{d^2 c_1}{dw d\sigma^2} \approx \frac{1}{8} \left[\frac{u''''(c)}{u''(c)} - \frac{u''''(c)u'''(c)}{u''(c)u''(c)} \right] \quad (10)$$

where $c = c_1 = \mathbb{E}c_2 = 1$. Equation (10) is positive if and only if the utility function has the property decreasing absolute prudence $\frac{d(-\frac{u''''}{u''})}{dc} < 0$.

The marginal consumption response to wage changes is

$$\frac{dc_1}{dw} = \frac{\mathbb{E}\{u''(c_2)\}}{u''(c_1) + \mathbb{E}\{u''(c_2)\}}.$$

There are two channels through which income risk affects this expression. First, when income risk increases, the household saves more for precautionary reasons. This precautionary saving raises consumption in period two relative to period 1. Through this channel, $\mathbb{E}u''(c_2)$ increases while $u''(c_1)$ decreases since $u''' > 0$. The total effect of this channel is negative and described by the second term in (10). Second, income risk itself reduces $\mathbb{E}\{u''(c_2)\}$ if it is concave ($u'''' < 0$) by Jensen's inequality. This channel strengthens the current consumption response to wage changes and is described by the second term in (10).

Proposition 2 further shows that (10) is positive if and only if the utility function satisfies decreasing absolute prudence. This result follows immediately because (10) is equivalent to the definition of decreasing absolute prudence.

Corollary 2. *Assume that $u'(c) = c^{-\gamma}$ with $\gamma > 0$, then (10) becomes*

$$\frac{d^2 c_1}{dw d\sigma^2} \approx \frac{1}{8}(1 + \gamma) > 0. \quad (11)$$

Since the power utility function has the property decreasing absolute prudence, it follows from Corollary 2 that the marginal effect of income risk on the current consumption response to wage changes is positive. Since the consumption response to wage changes is positive, it follows that higher income risk *strengthens* the consumption response to wage changes.⁵

2.4 Taking Stock

The main takeaways from this section are that income risk affects both how households respond to interest rate and wage changes. Income risk dampens the current consumption response to interest rate changes, while it amplifies the current consumption response to wage changes as long as preferences satisfy decreasing absolute prudence. Since monetary policy affects both the interest rate and the wage through general equilibrium effects, the effect of income risk on monetary transmission is ambiguous. The following sections explore how income risk affects monetary transmission in more realistic quantitative models.

3 Partial Equilibrium

The analytical results in Section 2 hold only in a two-period model where income risk is approximately zero. Although the results help expose the channels at work and the sign of the effects, we are ultimately interested in more general settings where income risk is strictly positive. This section simulates a consumption-saving model to explore to what extent the theoretical channels may be relevant for monetary transmission in a quantitative model.

⁵The results in Proposition 2 and Corollary 2 can be viewed as a special case of [Carroll and Kimball \(1996\)](#) who show that introducing income risk to a standard consumption problem results in a concave consumption function (and thus higher marginal propensity to consume) if the utility is HARA (hyperbolic absolute risk aversion).

3.1 The Model

An infinitely-lived household maximizes the utility flow from consumption and faces idiosyncratic income risk. The household problem can be formulated as

$$\max_{\{c_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c) dt \quad (12)$$

subject to

$$\begin{aligned} da_t &= (r_t a_t + w_t z_t - c_t) dt \\ dz_t &= -\nu(z_t - \bar{z}) dt + \sigma_t dW_t \\ a_t &\geq 0 \end{aligned}$$

where ρ is the discount rate, z is idiosyncratic income, $\nu = -\log(\theta)$ is the mean-reversion for an annual autocorrelation of θ , and σ is the standard deviation of the income process. There are three differences relative to the analytical results in Section 2: the agent is infinitely-lived, the income process is now an Ornstein-Uhlenbeck process in levels, and risk is strictly positive in the simulations. While non-standard, the assumption of an Ornstein-Uhlenbeck in levels ensures that a change in risk does not affect mean income and thus isolates the effect of risk in a similar way as in Section 2.⁶

Calibration. The parameters of the model are calibrated to standard values in the literature. The coefficient of relative risk aversion is 2, implying an elasticity of intertemporal substitution (EIS) of 1/2. The real interest rate is 0.04, and the time discount rate is 0.041. The borrowing constraint is the natural borrowing constraint. The income process is calibrated to match the results in Storesletten et al. (2004) where the process is normalized ($\mathbb{E}(z) = 1$, $w = 1$) such that the scale of σ is in percentage deviation from mean income.

3.2 Simulations

This section presents simulated consumption responses to interest rate and wage changes for low and high-risk scenarios. The low-risk scenario displays impulse responses in the benchmark calibration. On the other hand, the high-risk scenario displays impulse responses along the transition path of a permanent income risk shock.⁷ The path of realized productivity is fixed and constant in both scenarios, and thus not affected by the change in income risk. The difference

⁶Figure A.6 and A.7 display the risk effect in a setting where the Ornstein-Uhlenbeck process is specified in logs.

⁷The high-risk response is constructed as the difference between two paths: the response to an interest rate (or wage) shock and a permanent income risk shock relative to the responses to a permanent income risk shock alone.

	Value	Description	Target
γ	2	CRRA parameter, inverse EIS	Standard
r	0.04	Annual real interest rate	Standard
ρ	0.041	Time-discount rate	Standard
w	1	Wage	Normalization
θ	0.96	Annual autocorrelation of income	Storesletten et al. (2004)
σ	0.12	sd of income process in expansion	Storesletten et al. (2004)

Table 1: Model calibration

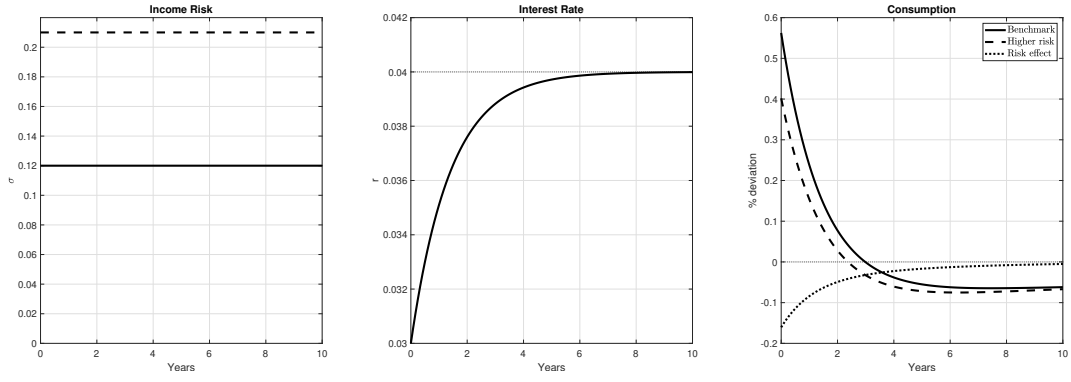
between the responses in the two cases thus isolates how a change in income risk affects the policy functions of individuals.

All simulations show the responses of one individual agent starting with mean income and zero wealth. Income equal to mean income ensures that the drift of income is zero such that the households do not expect higher or lower income in the future (in the policy function). Zero wealth ensures that there is no income effect of interest rate changes such that the model mimics the setting in Section 2. The following paragraphs present how the individual agent responds to interest rate and wage changes.

Interest rate changes. The interest rate shock considered is an unexpected rate cut that mean-reverts with an annual autocorrelation of 0.5. The model assumes that the interest rate shock is a zero probability event (“MIT”-shock). Figure 1 presents income risk, interest rate, and consumption responses for one individual agent starting with mean income and zero wealth in two scenarios: low risk and high risk. The difference between the low risk and high-risk responses is the isolated impact of higher income risk on how households respond to an interest rate shock, depicted as a dotted line. The increase in income risk corresponds to the permanent change in income risk from 0.12 to 0.21 between expansions and recessions (Storesletten et al., 2004).⁸

In Figure 1, the household initially responds to the interest rate cut by increasing consumption. This extra spending is financed by running down savings. As the interest rate eventually reverts back to its mean, wealth and capital income is lower, and the household keeps consumption low for a long period thereafter. The consumption response is similar in the high-risk scenario, albeit weaker. The dotted line displays the isolated effect of higher risk on the consumption response (the risk effect), defined as the difference between the consumption response in the high risk and the low-risk scenario. The risk effect is negative and significant. Higher risk dampens the consumption response to the interest rate cut by around 1/3 in the first year after the interest rate

⁸Since the paper considers changes in income risk at the business cycle frequency, the assumption that the income risk shock is permanent may seem unreasonable. Therefore, the results should be interpreted as an upper bound to the potential effect a change in risk may have on interest rate sensitivity.

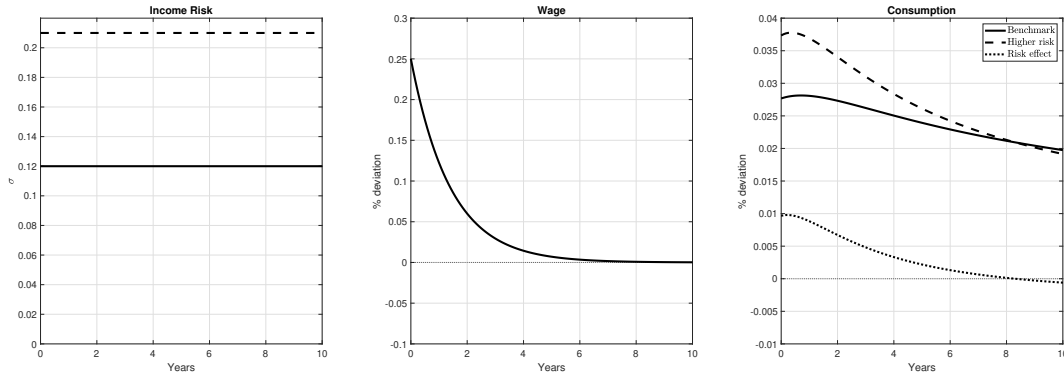


Notes: The figure shows the impulse responses to a quarter of a percentage point cut in the interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the interest rate shock.

Figure 1: Impulse responses to a interest rate cut.

cut.

Wage changes. Figure 2 considers the household's responses to an unexpected 0.25 percent increase in the wage that mean-reverts with an annual autocorrelation of 0.5. Again, the model assumes that the interest rate shock is a zero probability event ("MIT"-shock). As in Figure 1, Figure 2 also compares impulse responses under two scenarios: low-risk and high-risk.



Notes: The figure shows the impulse responses to a 0.25 percent increase in the wage that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the wage increase.

Figure 2: Impulse responses to a wage increase

Figure 2 shows that the household responds to the wage increase by increasing consumption. Since the wage rise is transitory, the household saves some additional earnings to smooth consumption over time. As the wage eventually reverts to its steady-state value, the household is wealthier, allowing it to keep consumption heightened for an extended period.

The dotted line illustrates the risk effect. It is defined as the difference between the responses in the high-risk and the low-risk cases. The risk effect is positive and significant. Higher income risk amplifies the consumption response to wage changes by around 1/3 in the first year after the wage change.

The Online Appendix contains several robustness exercises to verify that the results in this section do not depend on the specific calibration. First, Figure A.1 and A.2 show that the results are symmetric in the sense that a negative income risk shock has similar effects on interest rate and wage sensitivity with opposite signs. Second, Figure A.3 illustrates that the risk effects on interest rate and wage sensitivity are close to linear. Hence, the risk effect scales approximately linearly with the size of the risk change. Third, Figure A.4 and A.5 show that the risk effect is the same when considering half as large interest rate and wage shocks. Hence, the relative size of the risk effect does not depend on how large the interest rate or wage response is. Fourth, Figure A.6 and A.5 consider the case where the income process is specified in logs rather than in levels. With log income, a change in income risk also affects the drift of the income process. Hence, the risk shock simultaneously affects both the mean and variance of income innovations. The two

figures illustrate that the risk effect is similar also in the case with a log income process. However, the risk effect is smaller, dampening interest rate sensitivity by around 28% and amplifying wage sensitivity by 19%.

3.3 Taking Stock

The results in this section reveal that income risk significantly affects an individual's responses to interest rate and wage changes. While income risk dampens the consumption response to interest rate changes, it amplifies the responses to wage changes. Since a change in monetary policy both affects the interest rate and the wage, the two risk effects may partially cancel each other out in response to a monetary policy shock. The following section explores how income risk affects monetary transmission in a general equilibrium model where interest rates and wages move in response to the monetary policy shock.

4 General Equilibrium

This section analyzes the effects of income risk on monetary transmission in a heterogeneous-agent New-Keynesian model with an explicit role for monetary policy. In contrast to the two previous sections, monetary policy is endogenous and general equilibrium feedback effects exist.

4.1 The Model

The model is a one-asset heterogeneous agent New-Keynesian model. Households are similar to Section 3.1 with two exceptions. First, income now follows an Ornstein-Uhlenbeck in logs; and second, households face a labor supply choice. The firm-side is standard for New-Keynesian models where firms face sticky prices and compete under monopolistic competition. Households can save in an inelastically supplied risk-free bond. The nominal interest rate follows a Taylor rule.

Households. There exists a continuum of ex ante identical households facing idiosyncratic income risk and a labor supply choice. They solve the following problem

$$\max_{\{c_t, l_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c, l) dt \quad (13)$$

subject to

$$\begin{aligned} da_t &= (r_t a_t + w_t e^{z_t} l_t + div_t - tax_t - c_t) dt \\ dz_t &= -\nu(z_t - \bar{z}) dt + \sigma_t dW_t \\ a_t &\geq 0 \end{aligned}$$

where l_t is the supply of labor by the individual, div_t is profits from owning the intermediate goods firms (distributed equally across households), tax_t is lump-sum taxes, and $\nu = -\log(\theta)$ is the mean-reversion for an annual autocorrelation equal to θ . Further, the utility function is

$$u(c_t, l_t; z_t) = \frac{\left(c_t - e^{z_t} \frac{l_t^{1+1/\phi}}{1+1/\phi}\right)^{1-\gamma}}{1-\gamma} \quad (14)$$

where ϕ is the Frisch elasticity. Labor supply is given as

$$l_t = w_t^\phi,$$

which implies that labor supply only depends on the aggregate wage and not on any of the idiosyncratic states (a, z) . The specific form ensures that the household problem is exactly the same as in the two previous sections except that households respond to the aggregate wage.

Final good firm. There is a representative final good firm that produces final goods with a CES-aggregator

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the elasticity of substitution between different consumption goods.

Intermediate goods firms. There exists a continuum of intermediate goods firms that produce intermediate goods using a linear technology with labor as only input

$$y_{j,t} = n_{j,t}$$

where $n_{j,t}$ is the amount of effective labor units working for firm j . They compete under monopolistic competition with sticky prices. The specific form of Rotemberg-pricing from [Kaplan et al. \(2018\)](#) is adopted, resulting in the following New Keynesian Phillips curve

$$\left(r_t - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\varepsilon}{\psi} (\mu_t - \mu^*) + \pi_t$$

where $\mu^* = \frac{\varepsilon-1}{\varepsilon}$ is the marginal cost in steady state. All households own an equal share of the intermediate goods firms.

Monetary policy. The monetary policy authority sets the nominal interest rate according to a Taylor rule

$$\dot{i}_t = \bar{r} + \phi_\pi \pi_t + \eta_t$$

where ϕ_π is the Taylor coefficient on inflation and η_t is the monetary policy shock.

Government. The government collects lump-sum taxes from households and pays interest on bonds. It runs a balanced budget and adjusts taxes in response to changes in the real interest rate.

$$tax_t = r_t A$$

Equilibrium. The equilibrium is described by paths of aggregate variables $\{C_t, A_t, N_t, Y_t, \Pi_t, F_t\}_{t=0}^{\infty}$, individual variables $\{c_t, a_t, z_t, l_t\}_{t=0}^{\infty}$, and prices $\{r_t, w_t, \pi_t\}_{t=0}^{\infty}$ that satisfy the following:

1. *Households optimize:* Taking as given the interest rate and wage in the economy, households maximize utility subject to the budget constraint, the liquidity constraint, and the income process.
2. *Firms optimize:* Firms maximize profits subject to individual demand functions and price stickiness costs.
3. *Market clearing:* The goods, labor, and bond markets clear for all t ,

$$\begin{aligned} C_t + \kappa_t &= Y_t \\ N_t &= \int l_t(a, z) z_t(a, z) dF_t(a, z) \\ A_t &= A \end{aligned}$$

where κ_t is the total cost for price changes for all firms.

4. *Aggregation:* Aggregate quantities are consistent with the distribution of households

$$C_t = \int c_t(a, z) dF_t(a, z) \qquad A_t = \int a_t(a, z) dF_t(a, z)$$

where $F_t(a, z)$ is the joint distribution of wealth and income at time t .

4.2 Calibration

The parameters are calibrated to standard values in the literature. The discount rate ρ is set to match an annual wealth-to-income ratio of 4. The elasticity of intertemporal substitution and real interest rate are the same as in Section 3. The income process is calibrated to match Storesletten et al. (2004). The Frisch elasticity is set to 0.5, in the range of estimates found in Chetty, Guren, Manoli, and Weber (2011). The elasticity of substitution between different goods is set to 10 to match a profit share of around 10 percent. The price stickiness parameter ψ is set to 100 so that the Phillips curve slope of 0.1.

	Value	Description	Target
γ	2	CRRA parameter, inverse EIS	Standard
ϕ	0.5	Frisch elasticity	Standard
r	0.04	Annual real interest rate	Standard
ρ	0.045	Discount rate	Wealth/income = 4
θ	0.96	Annual autocorrelation of income	Storesletten et al. (2004)
σ	0.12	sd of income process in expansions	Storesletten et al. (2004)
$\bar{z}$	0.925	Income process parameter	Normalize such that $E(z) = 1$
ε	10	Elasticity of substitution in $y_{j,t}$	Profit share of 10%
ψ	100	Price stickiness parameter	Slope of Phillips curve, $\varepsilon/\psi = 0.1$

Table 2: Calibration of the general equilibrium model

4.3 The Income Risk Effect

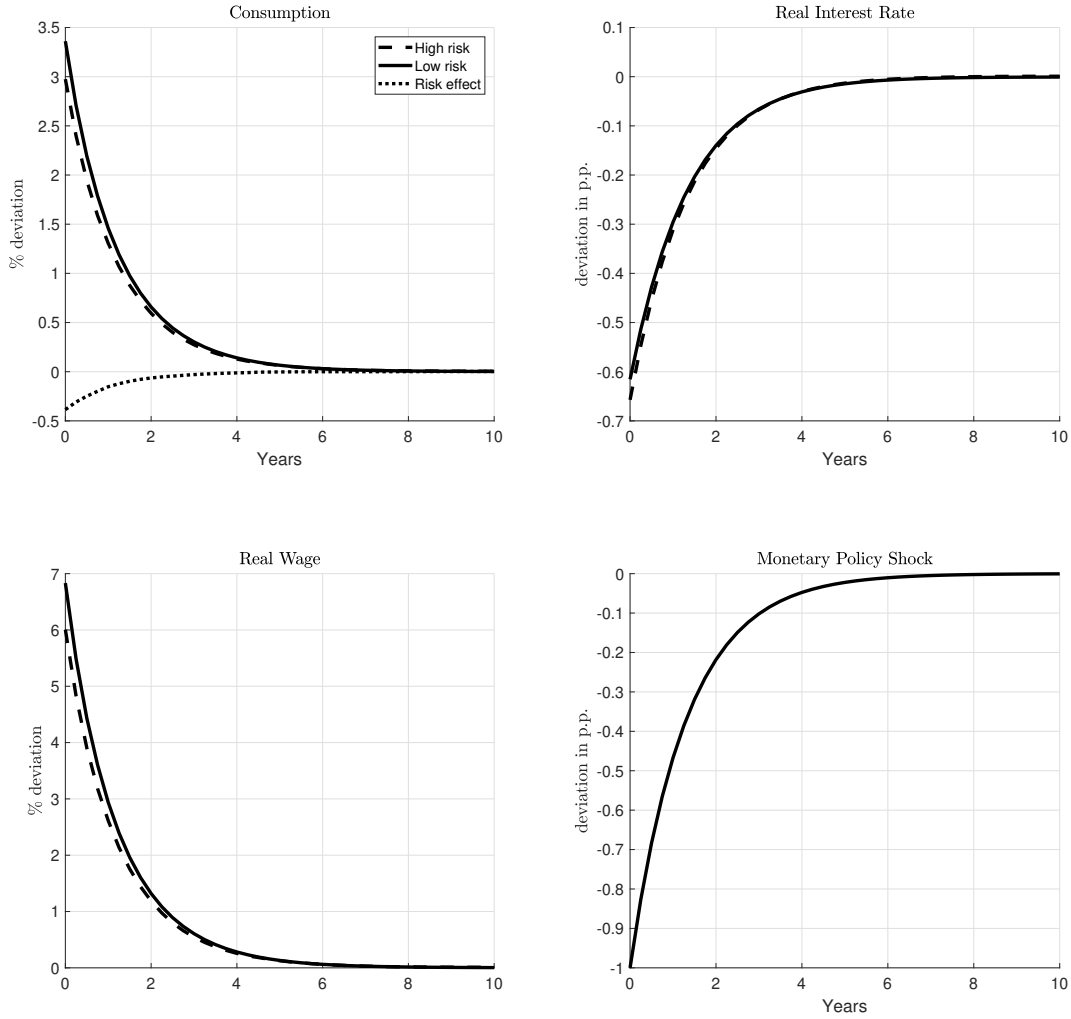
The key result concerns households' responses to an unexpected monetary policy shock that reduces the real interest rate by one percentage point and reverts to the mean with an annual autocorrelation of 0.5. The model assumes that the monetary policy shock is a zero probability event ("MIT"-shock). Figure 3 compares the responses for two different scenarios: low and high risk. In the low-risk scenario, income risk is kept constant at 0.12. In contrast, the high-risk scenario depicts the effect of monetary policy along a transition path from a permanent shock to income risk.⁹ The exercise is the same as in Section 3 where all parameters and the level of resources for each individual are the same, but income risk is different. Furthermore, the simulation considers the case where only perceived income risk increases, but not where it affects actual income innovations.¹⁰ The difference between the high risk and the low-risk responses, therefore, illustrates the isolated impact of higher perceived income risk on monetary transmission.

Figure 3 presents the impulse responses. Households initially increase consumption after the interest rate reduction. Firms respond to this increase in aggregate demand by raising wages to attract more labor. As wages increase, households experience higher income and further increases consumption. The central bank reacts to higher inflation by raising the interest rate, dampening somewhat the initial impact of the monetary policy shock.

The main question is how income risk affects monetary transmission to aggregate consumption. In Figure 3, the solid lines depict the responses without any change in income risk, while the dashed lines show the responses along the transition path under a permanent increase in income

⁹Numerically, the permanent change in income risk is implemented as a shift in income risk that reverts back after 30 years. The impulse responses presented are insensitive to the exact choice of 30 years as long as it is sufficiently far into the future.

¹⁰The case where actual income risk also increases is presented in Figure A.8 in the Online Appendix and discussed below.



Notes: The figure shows the aggregate impulse responses to a one percentage point cut in the nominal interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the high-risk and low-risk responses, and illustrates the effect of more income risk on the consumption response to the monetary policy shock.

Figure 3: Impulse responses to an expansionary monetary policy shock

risk. The dotted line is the difference between the high-risk and the low-risk case and illustrates how heightened risk affects the responses. The risk effect is negative and small. A 75% increase in the standard deviation of income, consistent with the difference in income risk between expansions and recessions (Storesletten et al., 2004), dampens monetary transmission by around 11%.

Since the results in Section 3 show that higher risk weakens interest rate sensitivity but strengthens wage sensitivity, it is not surprising that higher perceived risk has a relatively weak effect on monetary transmission. The aggregate weakening of monetary transmission is also consistent with the observation in Figure A.6 and A.7 that when the income process is specified in logs, a change in risk has a more significant impact on interest rate sensitivity than on wage sensitivity.

A dampening of monetary transmission by 11% is relatively small. However, there are two additional reasons for arguing that higher income risk has a relatively weak impact on monetary transmission. First, the simulation in Figure 3 illustrates the impact of risk in the case where the change in risk is permanent. However, since income risk varies over the business cycle, it is perhaps more reasonable to consider it as a mean-reverting process. And if the increase in income risk is less persistent, its impact on monetary transmission is also weaker. Second, Figure 3 only considers the case where *perceived* income risk increases. While one can argue that perceived income risk increases more than actual income risk in recessions, at least part of the change in income risk will also affect households' income. Figure A.8 illustrates the case where actual income risk increases from 0.12 to 0.21. In this case, the effect of heightened risk is to *strengthen* monetary transmission, the opposite of what it is in Figure 3, although the effect is still small. Since a change in risk dampens the risk effect relative to that for perceived risk, one can argue that the effect of perceived risk in Figure 3 overstates the total effect of risk on monetary transmission.

The results in Figure 3 relies on one specific calibration of the model. In particular, Corollary 1 and 2 show that both the marginal effect of risk on the interest rate and wage response depends on γ , either representing the elasticity of intertemporal substitution or prudence (strength of precautionary saving motive). The Online Appendix contains results on the size of the risk effect under two alternative calibrations: $\gamma = 1.5$ (Figure A.9) and $\gamma = 2.5$ (Figure A.10). The size of the risk effect increases with γ , increasing from 8% when $\gamma = 1.5$, to 11% in the benchmark case with $\gamma = 2$, to 14% when $\gamma = 2.5$. However, while the size of the risk effect varies across calibrations, it remains relatively small for standard values of γ .

5 Conclusion

This paper asks to what extent changes in perceived income risk affect monetary transmission. It first investigates the channels through which income risk affects monetary transmission to non-durable consumption in a partial equilibrium consumption-saving model. While income risk weakens the substitution effect from interest rate changes, it strengthens the indirect income effect from wage changes. Furthermore, the effects of income risk are potentially sizable, affecting households' responses to interest rates and wages by around 1/3.

Section 4 next investigates how income risk affects monetary transmission in a heterogeneous agent New-Keynesian model. In this model, monetary policy is endogenous, and there are general equilibrium feedback effects. Consistent with the theoretical results, an increase in income risk affects monetary transmission to aggregate consumption. However, the total effect is small because the channels through which income risk affects monetary transmission partially cancel each other out. In the benchmark calibration, an increase in income risk consistent with the estimates from Storesletten et al. (2004) on business cycle variation in risk weakens monetary transmission by around 11%. Hence, while income risk significantly affects individuals' responses to interest rate

and wage changes, its impact on aggregate monetary transmission is small.

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Online Appendix

A Proofs

A.1 Proof of Lemma 1

Proof. The Euler equation is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'((1+r)(w-c_1) + 1 + \zeta)\}.$$

Compute the total derivative with respect to r

$$u''(c_1)\frac{dc_1}{dr} = \beta\mathbb{E}\{u'(c_2)\} + \beta(1+r)\mathbb{E}\{u''(c_2)\}[w-c_1 - (1+r)\frac{dc_1}{dr}]$$

which is (5) if $\beta = 1, r = 0, w = 1$, and $\sigma^2 \rightarrow 0$. □

A.2 Proof of Lemma 2

Proof. The Euler equation is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'((1+r)(w-c_1) + 1 + \zeta)\}.$$

Compute the total derivative with respect to w

$$u''(c_1)\frac{dc_1}{dw} = \beta(1+r)\mathbb{E}\{u''(c_2)\}\left[(1+r)\left(1 - \frac{dc_1}{dw}\right)\right]$$

which is (6) if $\beta = 1, r = 0, w = 1$, and $\sigma^2 \rightarrow 0$. □

A.3 Proof of Lemma 3

Proof. The Euler equation is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'(c_2)\}.$$

Compute the second order Taylor expansion of the right hand side around $c_2 = \mathbb{E}c_2$

$$\mathbb{E}\{u'(c_2)\} \approx u'(\mathbb{E}c_2) + \frac{1}{2}u'''(\mathbb{E}c_2)\sigma^2.$$

Compute the total derivative of the new expression with respect to σ^2

$$u''(c_1)\frac{dc_1}{d\sigma^2} = \beta(1+r)u''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \beta(1+r)u''''(\mathbb{E}c_2)\sigma^2\frac{d\mathbb{E}c_2}{d\sigma^2} + \beta(1+r)\frac{1}{2}u'''(\mathbb{E}c_2).$$

which is (7) if $\beta = 1, r = 0, w = 1$, and $\sigma^2 \rightarrow 0$. □

A.4 Proof of Proposition 1

Proof. The Euler equation is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'((1+r)(w-c_1) + 1 + \zeta)\}.$$

Compute the derivative with respect to r

$$u''(c_1)\frac{dc_1}{dr} = \beta\mathbb{E}\{u'(c_2)\} + \beta(1+r)\mathbb{E}\{u''(c_2)\}\left[w - c_1 - (1+r)\frac{dc_1}{dr}\right] \quad (15)$$

The second-order Taylor expansions of $E\{u'(c_2)\}$ and $E\{u''(c_2)\}$ are

$$\begin{aligned} E\{u'(c_2)\} &\approx u'(\mathbb{E}c_2) + \frac{1}{2}u'''(\mathbb{E}c_2)\sigma^2 \\ E\{u''(c_2)\} &\approx u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2 \end{aligned}$$

which implies that (15) becomes

$$u''(c_1)\frac{dc_1}{dr} \approx \beta u'(\mathbb{E}c_2) + \beta\frac{1}{2}u'''(\mathbb{E}c_2)\sigma^2 + \beta(1+r)\left(u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\right)\left[w - c_1 - (1+r)\frac{dc_1}{dr}\right]. \quad (16)$$

Compute the derivative with respect to σ^2 then gives

$$\begin{aligned} u'''(c_1)\frac{dc_1}{dr}\frac{dc_1}{d\sigma^2} + u''(c_1)\frac{d^2c_1}{drd\sigma^2} &\approx \beta u''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \beta\frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\frac{d\mathbb{E}c_2}{d\sigma^2} + \beta\frac{1}{2}u'''(\mathbb{E}c_2) \\ &+ \beta(1+r)\left(u'''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u''''(\mathbb{E}c_2)\right)\left[w - c_1 - (1+r)\frac{dc_1}{dr}\right] \\ &- \beta(1+r)\left(u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\right)\left(\frac{dc_1}{d\sigma^2} + (1+r)\frac{d^2c_1}{drd\sigma^2}\right) \end{aligned}$$

which evaluated at $\beta = 1, r = 0, w = 1$, and $\sigma^2 \rightarrow 0$ becomes

$$\begin{aligned} u'''(c_1)\frac{dc_1}{dr}\frac{dc_1}{d\sigma^2} + u''(c_1)\frac{d^2c_1}{drd\sigma^2} &\approx u''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u'''(\mathbb{E}c_2) \\ &- \left(u'''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u''''(\mathbb{E}c_2)\right)\frac{dc_1}{dr} - u''(\mathbb{E}c_2)\left(\frac{dc_1}{d\sigma^2} + \frac{d^2c_1}{drd\sigma^2}\right). \end{aligned}$$

Rearranging (note that since $\mathbb{E}c_2 = w - c_1 + 1$ and $w = 1$, then $\frac{dc_1}{d\sigma^2} = -\frac{d\mathbb{E}c_2}{d\sigma^2}$ and $\mathbb{E}c_2 = c_1 = c$)

$$\begin{aligned} \frac{d^2c_1}{drd\sigma^2} [2u''(c)] &\approx -u'''(c)\frac{dc_1}{dr}\frac{dc_1}{d\sigma^2} - u''(c)\frac{dc_1}{d\sigma^2} + \frac{1}{2}u'''(c) \\ &+ u'''(c)\frac{dc_1}{dr}\frac{dc_1}{d\sigma^2} - \frac{1}{2}u''''(c)\frac{dc_1}{dr} - u''(c)\frac{dc_1}{d\sigma^2}. \end{aligned}$$

Replace for $\frac{dc_1}{dr}$ and $\frac{dc_1}{d\sigma^2}$ using (5) and (7), respectively

$$\frac{d^2c_1}{drd\sigma^2} [2u''(c)] \approx -\frac{1}{4}\frac{u''''(c)u'(c)}{u''(c)}$$

which is the same as (8). □

A.5 Proof of Proposition 2

Proof. The Euler equation is

$$u'(c_1) = \beta(1+r)\mathbb{E}\{u'((1+r)(w-c_1)+1+\zeta)\}.$$

Compute the derivative with respect to w

$$u''(c_1)\frac{dc_1}{dw} = \beta(1+r)\mathbb{E}\{u''(c_2)\}\left[1 - \frac{dc_1}{dw}\right] \quad (17)$$

The second-order Taylor expansions of $E\{u''(c_2)\}$ is

$$E\{u''(c_2)\} \approx u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2$$

which implies that (17) becomes

$$u''(c_1)\frac{dc_1}{dw} \approx \beta(1+r)\left(u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\right)\left[1 - \frac{dc_1}{dw}\right]. \quad (18)$$

Compute the derivative with respect to σ^2 then gives

$$\begin{aligned} u'''(c_1)\frac{dc_1}{dw}\frac{d\sigma^2}{d\sigma^2} + u''(c_1)\frac{d^2c_1}{dw d\sigma^2} \\ \approx \beta(1+r)\left(u'''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u''''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2}\sigma^2 + \frac{1}{2}u''''(\mathbb{E}c_2)\right)\left[1 - \frac{dc_1}{dw}\right] \\ - \beta(1+r)\left(u''(\mathbb{E}c_2) + \frac{1}{2}u''''(\mathbb{E}c_2)\sigma^2\right)\frac{d^2c_1}{dw d\sigma^2} \end{aligned}$$

which evaluated at $\beta = 1$, $r = 0$, $w = 1$, and $\sigma^2 \rightarrow 0$ becomes

$$\begin{aligned} u'''(c_1)\frac{dc_1}{dw}\frac{d\sigma^2}{d\sigma^2} + u''(c_1)\frac{d^2c_1}{dw d\sigma^2} \\ \approx \left(u'''(\mathbb{E}c_2)\frac{d\mathbb{E}c_2}{d\sigma^2} + \frac{1}{2}u''''(\mathbb{E}c_2)\right)\left[1 - \frac{dc_1}{dw}\right] \\ - u''(\mathbb{E}c_2)\frac{d^2c_1}{dw d\sigma^2} \end{aligned}$$

Rearranging (note that since $\mathbb{E}c_2 = w - c_1 + 1$ and $w = 1$, then $\frac{dc_1}{d\sigma^2} = -\frac{d\mathbb{E}c_2}{d\sigma^2}$ and $\mathbb{E}c_2 = c_1 = c$)

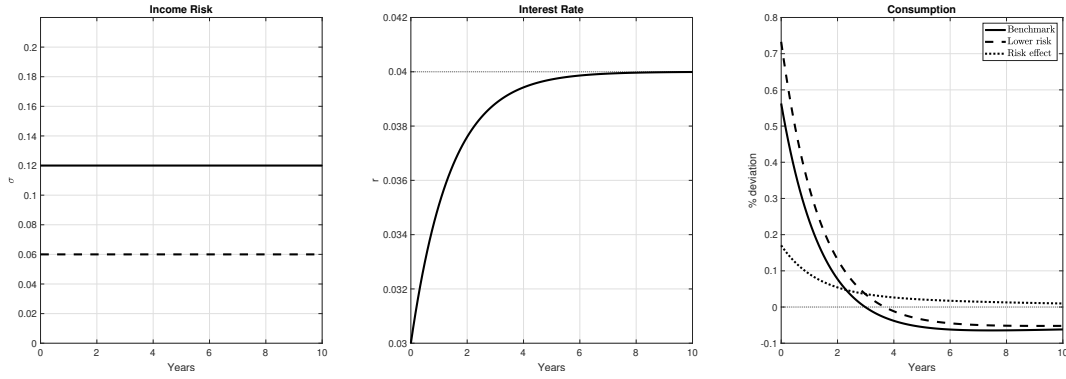
$$2u''(c)\frac{d^2c_1}{dw d\sigma^2} \approx \frac{1}{2}u''''(c)\left[1 - \frac{dc_1}{dw}\right] - u'''(c)\frac{dc_1}{d\sigma^2}.$$

Replace for $\frac{dc_1}{dw}$ and $\frac{d^2c_1}{d\sigma^2}$ using (6) and (7), respectively

$$2u''(c)\frac{d^2c_1}{dw d\sigma^2} \approx \frac{1}{4}u''''(c) - \frac{1}{4}\frac{u'''(c)u'''(c)}{u''(c)}.$$

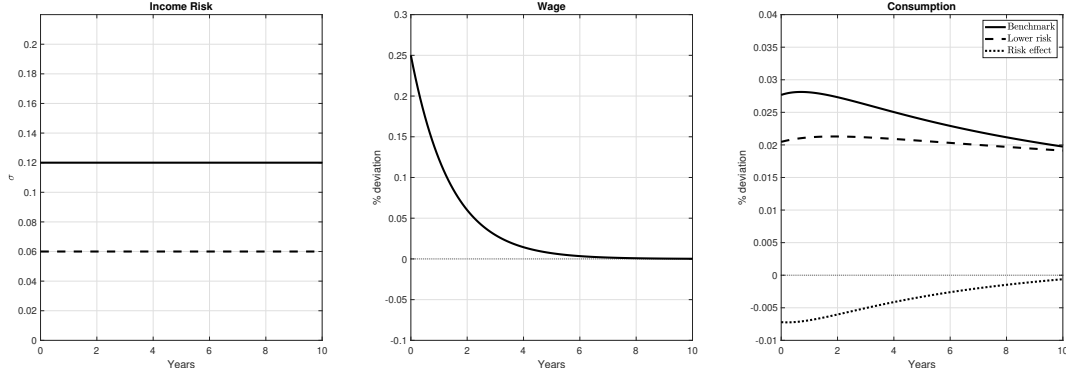
which is the same as (10). □

B Extra figures



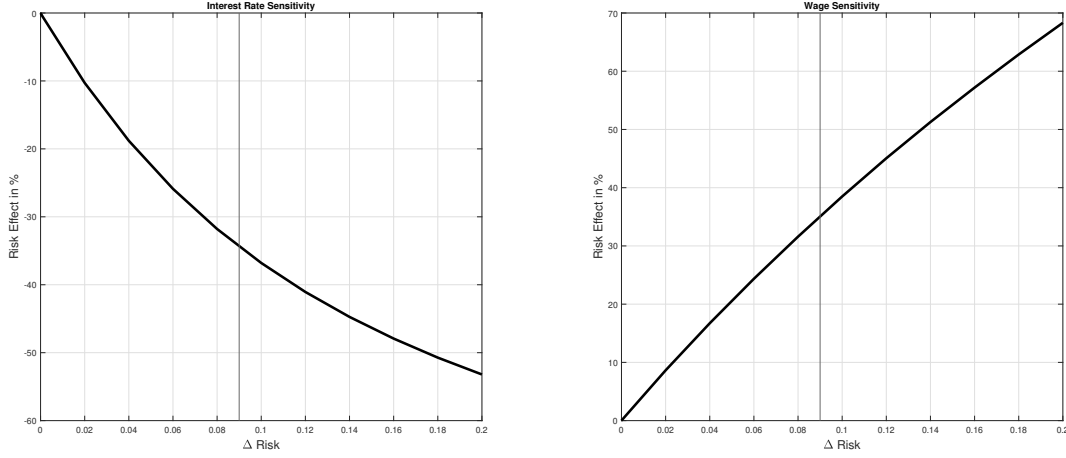
Notes: The figure shows the impulse responses to a quarter of a percentage point cut in the interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.06$. The black dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the interest rate shock.

Figure A.1: Impulse responses to a interest rate cut. A negative income risk shock.



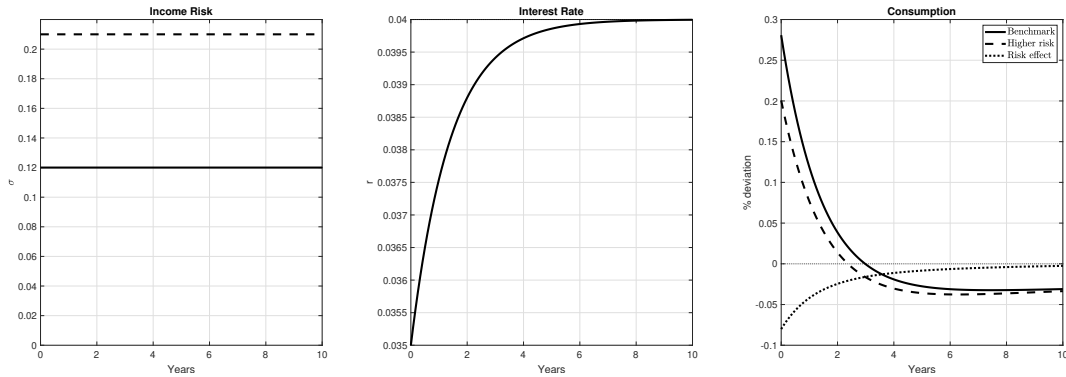
Notes: The figure shows the impulse responses to a 0.25 percent increase in the wage that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.06$. The dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the wage increase.

Figure A.2: Impulse responses to a wage increase. A negative income risk shock.



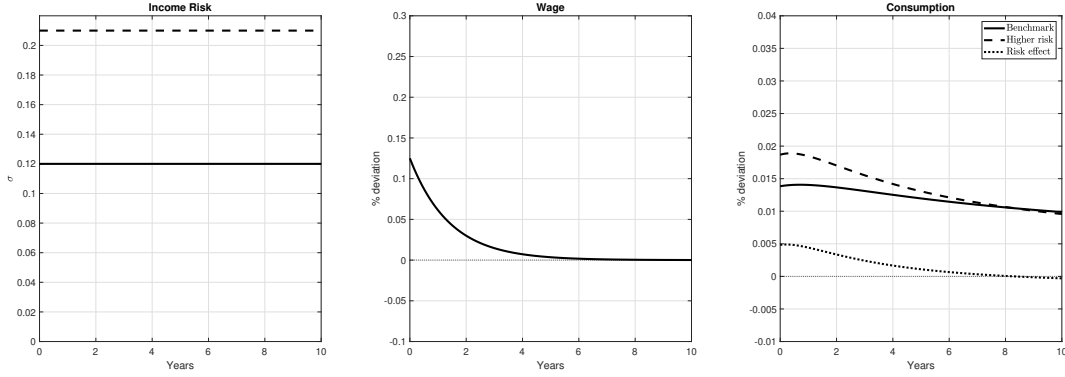
Notes: The figure shows the maximum impact of the risk change on the interest rate and wage responses by the change in income risk. The left figure is constructed by computing the difference between the consumption responses to a one percentage point interest rate cut that mean reverts at an annual rate of 0.5 in the benchmark case and along the transition path in the new risk case. The right figure is constructed by computing the difference between the consumption responses to a 0.25 percent wage increase that mean reverts at an annual rate of 0.5 in the benchmark case and along the transition path in the new risk case. The solid line at 0.09 is the change in income risk considered in the body of the paper.

Figure A.3: Risk effects by change in income risk



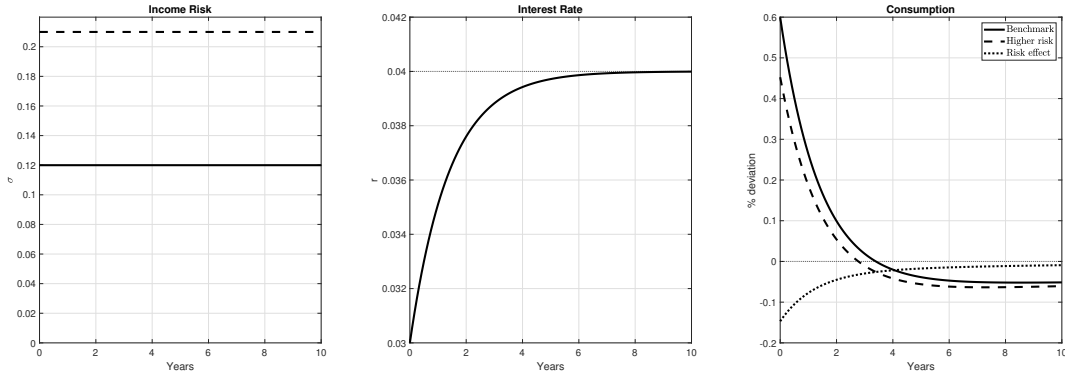
Notes: The figure shows the impulse responses to a quarter of a one half percentage point cut in the interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the interest rate shock.

Figure A.4: Impulse responses to a interest rate cut. Half as large interest rate cut.



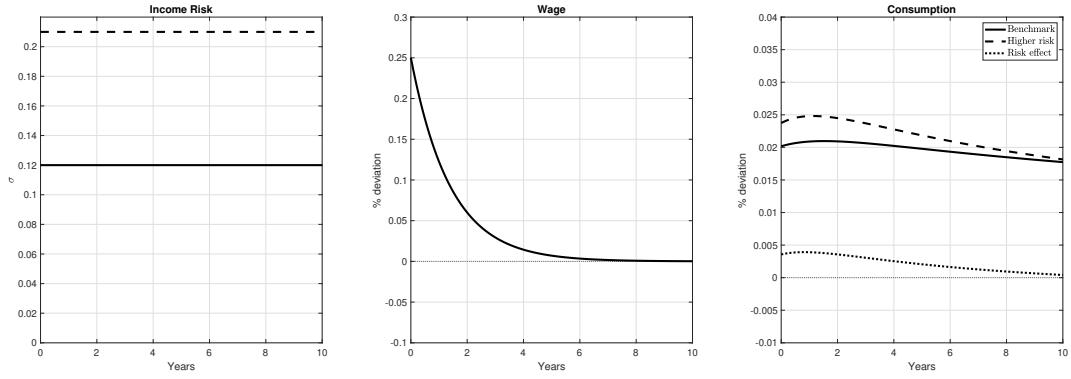
Notes: The figure shows the impulse responses to a 0.125 percent increase in the wage that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the wage increase.

Figure A.5: Impulse responses to a wage increase. Half as large wage increase.



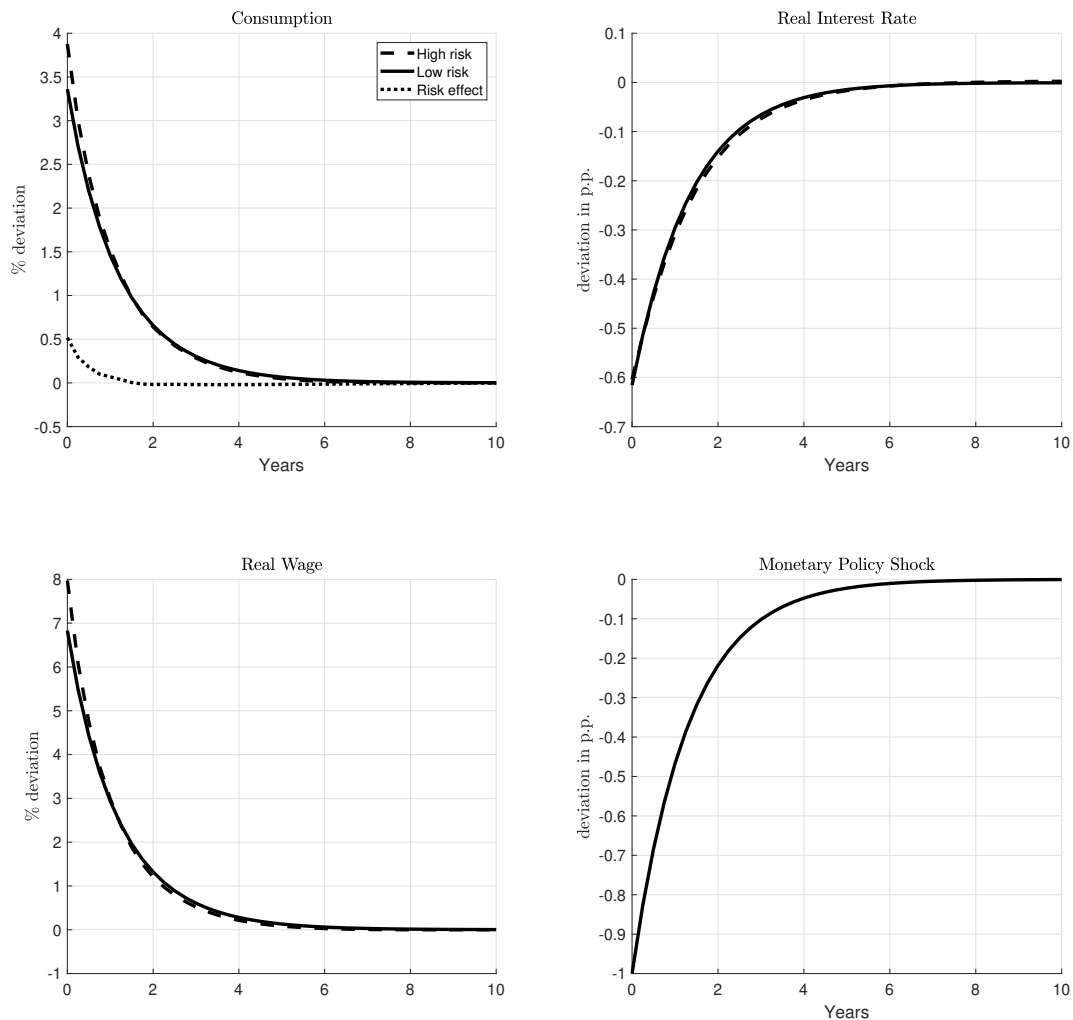
Notes: The figure shows the impulse responses to a quarter of a percentage point cut in the interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the interest rate shock. The only difference between this model and the model presented in Section 3 is that labor income is $w_t e^{z_t}$ when constructing this figure.

Figure A.6: Impulse responses to an interest rate cut. Income process in logs.



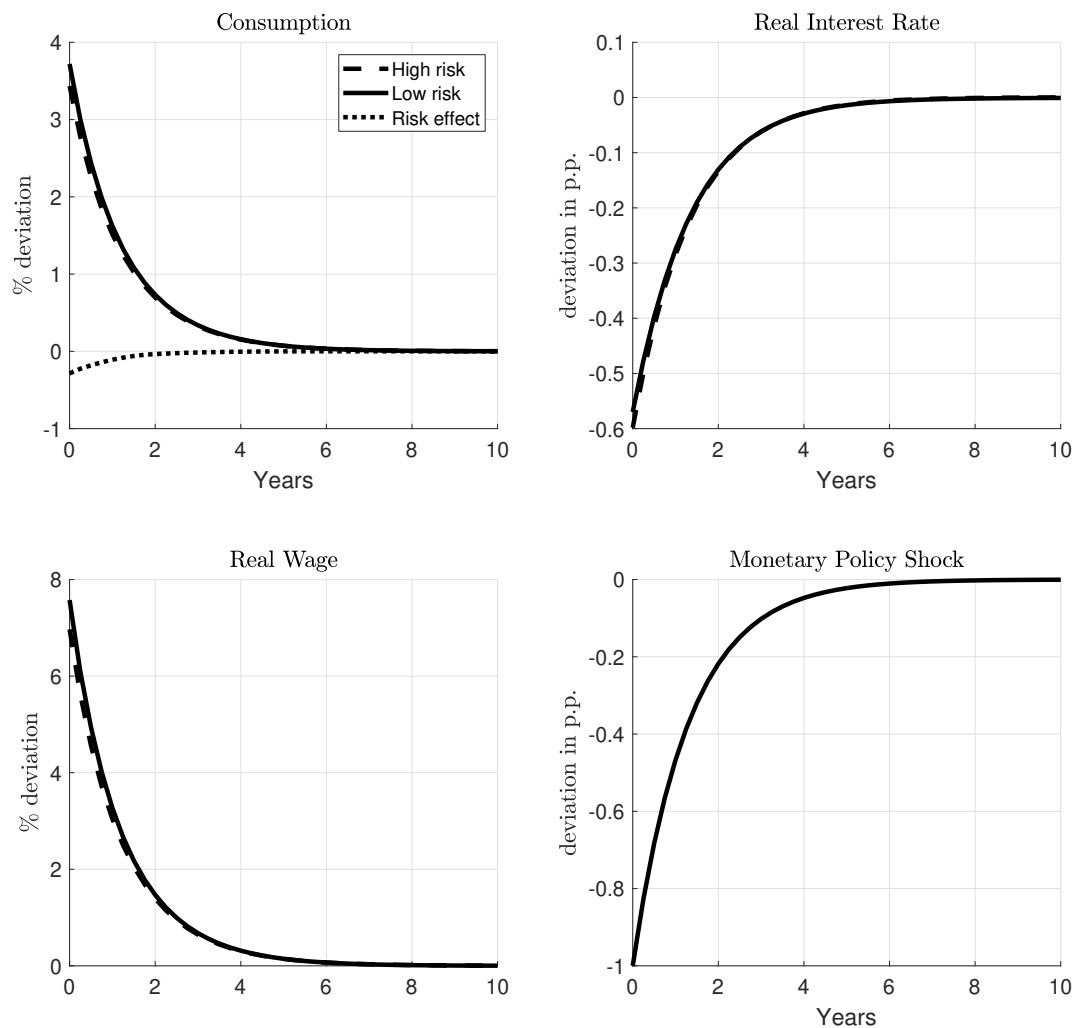
Notes: The figure shows the impulse responses to a 0.25 percent increase in the wage that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The dotted line is the difference between the dashed and the solid line (high-risk case - low-risk case) and illustrates the effect of more income risk on the consumption response to the wage increase. The only difference between this model and the model presented in Section 3 is that labor income is $w_t e^{z_t}$ when constructing this figure.

Figure A.7: Impulse responses to a wage increase. Income process in logs.



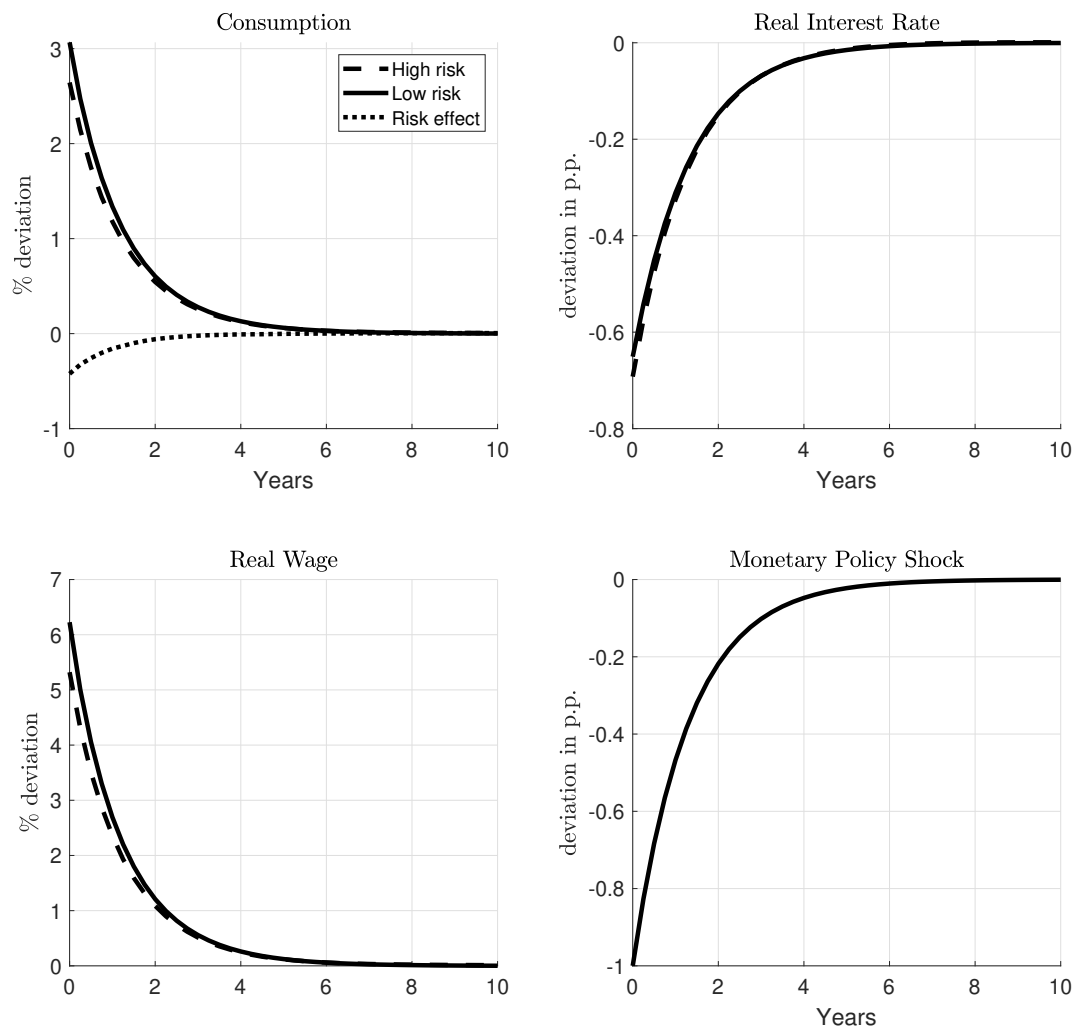
Notes: The figure shows the aggregate impulse responses to a one percentage point cut in the nominal interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the high-risk and low-risk responses, and illustrates the effect of more income risk on the consumption response to the monetary policy shock.

Figure A.8: Impulse responses to an expansionary monetary policy shock. Including changes in realized income innovations.



Notes: The figure shows the aggregate impulse responses to a one percentage point cut in the nominal interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the high-risk and low-risk responses, and illustrates the effect of more income risk on the consumption response to the monetary policy shock. This calibration has $\gamma = 1.5$.

Figure A.9: Impulse responses to an expansionary monetary policy shock. $\gamma = 1.5$.



Notes: The figure shows the aggregate impulse responses to a one percentage point cut in the nominal interest rate that mean-reverts at an annual rate of 0.5. The solid lines are the impulse responses from a calibration with $\sigma = 0.12$ while the dashed lines are the impulse responses along a transition path under a permanent income risk shock to $\sigma = 0.21$. The black dotted line is the difference between the high-risk and low-risk responses, and illustrates the effect of more income risk on the consumption response to the monetary policy shock. This calibration has $\gamma = 2.5$.

Figure A.10: Impulse responses to an expansionary monetary policy shock. $\gamma = 2.5$.